

ESSAYS on PROSPECT THEORY and ASSET PRICING

A Dissertation

Presented to the Faculty of the Graduate School

of Cornell University

In Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

by

Liyan Yang

May 2010

© 2010 Liyan Yang

ESSAYS on PROSPECT THEORY and ASSET PRICING

Liyan Yang, Ph. D.

Cornell University 2010

The financial markets are full of puzzles. In the aggregate market, stocks earn returns that cannot be justified by individual risk aversion (the equity premium puzzle); stock prices fluctuate much more than the underlying dividend process (the excess volatility puzzle); and stock returns can be predicted by many variables, such as dividend-to-price ratios or book-to-market ratios (the predictability puzzle). In the cross-section of stock returns, when stocks are sorted into different groups according to certain economic variables, including prior returns (the momentum puzzle), book-to-market ratio (the value premium puzzle), and size (the size puzzle), one group tends to earn higher average returns than another. At the individual trading level, a large body of evidence suggests that investors are reluctant to take losses (the disposition effect), tend to hold under-diversified portfolios (the under-diversification puzzle), and trade more than can be justified on rational grounds (the excessive trading puzzle). None of these facts can be explained by the traditional consumption-based asset pricing models; they are thus labeled as anomalies.

This study explores how models incorporating prospect theory preferences can improve our understanding of asset prices at both the aggregate market and individual stock levels. Chapter 1 studies a market-selection problem in an economy populated by Epstein-Zin investors and prospect theory investors. This chapter answers the questions of whether prospect theory investors can

survive and have price impact in the long run, and thus, this chapter lays down the foundation for using prospect theory preferences to understand financial markets. Chapter 2 examines the implications of prospect theory preferences for the disposition effect, the momentum effect in the cross-section of stock returns, and the correlation between returns and volumes. Chapter 3 first provides strong empirical evidence for volatility clustering in the dividend growth rate process and then incorporates this feature into an asset pricing model with prospect theory investors to explore its implications for the aggregate stock market.

BIOGRAPHICAL SKETCH

Liyan Yang graduated from Shandong University with a B.A. degree in economics in 1999. He continued his study in Shandong University and obtained his master degree in economics in 2002. In August of 2004, he began his doctoral studies in economics at Cornell University.

ACKNOWLEDGMENTS

I am grateful to my advisors, Lawrence Blume, David Easley, Ming Huang and Maureen O'Hara, for their kindness, support and guidance.

TABLE OF CONTENTS

BIOGRAPHICAL SKETCH	iii
ACKNOWLEDGMENTS	iv
TABLE OF CONTENTS	v
LIST OF FIGURES	vii
LIST OF TABLES	viii

CHAPTER ONE: MARKET SELECTION: LOSS AVERSION VS. EPSTEIN-ZIN PREFERENCES..... 1

1.1 Introduction	1
1.2 The Model	9
1.3 Equilibrium	14
1.3.1 Investors' Decisions	15
1.3.2 Stock Prices and Wealth Dynamics	20
1.4 Implications of Loss Aversion for Survival and Price Impacts	24
1.4.1 Representative-Agent Economy	25
1.4.2 EIS=1: Portfolio Selection	31
1.4.3 EIS \neq 1: Saving Behavior	41
1.5 Multi-Dimensional Heterogeneity in Preferences	48
1.5.1 Different EIS Parameters	50
1.5.2 Different Time Patience Parameter	52
1.6 Conclusion	54
Appendix	56

CHAPTER TWO: PROSPECT THEORY, THE DISPOSITION EFFECT AND ASSET PRICES 64

2.1 Introduction	64
2.2. The Model	70
2.2.1 Financial Assets	71
2.2.2 Beliefs	72
2.2.3 Preference	74
2.2.4 Timeline	76
2.2.5 Extension: A Multi-Stock Setting	78
2.3 Equilibrium	79
2.3.1 Age-3 Investors' Decisions	79
2.3.2 Age-2 Investors' Decisions	80
2.3.3 Age-1 Investors' Decisions	83
2.3.4 Evolution of State Variables	83
2.3.5 Market Clearing Condition	84
2.3.6 Benchmark Case: Standard Risk Neutral Utility	85

2.4 Numerical Results and Intuitions	86
2.4.1 Calibrating Technology Parameters	87
2.4.2 Implications of Diminishing Sensitivity.....	88
2.4.2.1 Disposition Effects	89
2.4.2.2 Momentum	94
2.4.2.3 Turnover	95
2.4.2.4 Equity Premiums	97
2.4.3 Implications of Loss Aversion.....	98
2.4.3.1 Reversed Disposition Effects	99
2.4.3.2 Reversal	103
2.4.3.3 Turnover	103
2.4.3.4 Equity Premiums	104
2.4.4 Quantitative Analysis and Testable Predictions	105
2.3.4.1 Quantitative Analysis: How Successful is Prospect Theory?.....	105
2.4.4.2 Testable Predictions.....	108
2.5 Conclusion	111
Appendix	112

CHAPTER THREE: DIVIDEND VOLATILITY and ASSET PRICING 126

3.1 Introduction.....	126
3.2. Key Features of Historical Dividend Volatility	131
3.2.1 Dividend Volatility Clustering	131
3.2.2 Time-Varying Dividend Volatility	136
3.3 The Model	141
3.3.1 Setup	141
3.3.2 Equilibrium Prices	146
3.3.3 Methodology of Numerical Computation	147
3.4 Model Results.....	147
3.4.1 Calibrating Parameter Values.....	147
3.4.2 Price-dividend Ratio Function	150
3.4.3 Simulation Results.....	154
3.4.3.1 Stock Returns and Stock Volatility	155
3.4.3.2 Autocorrelations of Returns and Price-Dividend Ratios	157
3.4.3.3 Return Predictability.....	158
3.4.3.4 Time-varying Sharpe Ratios.....	161
3.4.3.5 Structural Break and Equity Premiums	163
3.5 Conclusion.....	165
Appendix	167

LIST OF FIGURES

Figure 1.1: Survival and Price Impacts when $EIS_{EZ} = EIS_{LA} = 1$	35
Figure 1.2: Price-Impact Function and Investment Policy when $EIS_{EZ} = EIS_{LA} = 1$	39
Figure 1.3: Survival when. $EIS_{EZ} = EIS_{LA} = 1.5$	42
Figure 1.4: Consumption Policy and Risk-Free Rate when. $EIS_{EZ} = EIS_{LA} = 0.5$	46
Figure 1.5: Survival and Consumption Policies when when $b_{LA} = 0.2$ and $EIS_{EZ} = EIS_{LA} = 0.5$	48
Figure 1.6: Survival and Consumption Policies when $EIS_{EZ} < EIS_{LA}$	51
Figure 1.7: Survival and Consumption Policies when $\beta_{EZ} < \beta_{LA}$	53
Figure 2.1: Timeline.....	77
Figure 2.2: Diminishing Sensitivity Drives the Disposition Effect.....	92
Figure 2.3: Loss Aversion Drives the Reversed Disposition Effect.....	102
Figure 3.1: Dividend Volatility, GDP Growth and Recessions	138
Figure 3.2: Smoothed Probability of a High Volatility Regime and Historical Dividend Growth Rate.....	140
Figure 3.3: Gain and Loss Function	142
Figure 3.4: Price-Dividend Functions f	151
Figure 3.5: Historical Price-Dividend Ratios v.s. Conditional Dividend Volatility	154
Figure 3.6: Conditional Moments of Stock Returns	161
Figure 3.7: Distribution of the Conditional Sharpe Ratios.....	163

LIST OF TABLES

Table 1.1: Technology Parameter Values	25
Table 1.2: Asset Prices and Consumption Policies in Representative Agent Economies.....	27
Table 1.3: Survival and Price Impacts: EIS=1.	33
Table 1.4: Survival and Price Impacts: EIS=0.5.	44
Table 2.1 :Technology Parameter Values	88
Table 2.2: Implications of Diminishing Sensitivity	90
Table 2.3: Implications of Loss Aversion	100
Table 2.4:Quantitative Analysis	106
Table 2.5:Sensitivity Analysis w.r.t Dividend Growth Rate Volatility $\sigma(\theta_{t+1})$..	110
Table 2.A1:Results for a Decision Interval of Six Months	114
Table 2.A2:Results for Using Purchase Prices as Reference Points.....	116
Table 3.1:Dividend Volatility Estimates	132
Table 3.2:Calibrated Parameters.....	149
Table 3.3:Asset Prices and Annual Returns (1926-2006)	156
Table 3.4:Autocorrelations of Returns and Price-Dividend Ratios.....	158
Table 3.5:Return Predictability Regressions (1926-2006)	160
Table 3.6:Structural Break and Asset Prices	165

CHAPTER 1

MARKET SELECTION: LOSS AVERSION vs. EPSTEIN-ZIN PREFERENCES

1.1 Introduction

The traditional consumption-based asset-pricing approach assumes that agents' preferences are consistent with Savage's notion of subjective expected utility. As is well-known, when combined with rational expectation equilibrium, this approach has difficulty in closely matching the financial market data (e.g., Hansen and Singleton, 1983; Mehra and Prescott, 1985; and Hansen and Jagannathan, 1991). In response to this difficulty, researchers have used more realistic and sophisticated preference specifications to model people's behavior in an attempt to better understand the financial phenomena. Prospect theory, particularly the loss-aversion feature of prospect theory, is one of these new specifications and has proven itself as a successful complement to the expected-utility framework in explaining asset prices and trading behavior (See Barberis and Thaler, 2003). The literature has labeled this new approach relying on prospect theory as the loss aversion/narrow framing approach (Barberis and Huang, 2007, 2009).

However, most models adopting the loss aversion/narrow framing approach are conducted in a representative agent framework (e.g., Benartzi and Thaler, 1995; Barberis and Huang, 2001, 2007, 2009; Barberis, Huang and Santos, 2001; McQueen and Vorkink, 2004; Grünea and Semmler, 2008), which makes it very hard to interpret the results in these models. In reality, it is highly likely that investors who are not loss-averse trade against investors who are, thus attenuating their effects. This concern has led Barberis and Huang (2009,

p 1567) to caution that one should interpret the equity premium obtained in their representative-agent model as an upper bound on the equity premium that we would obtain in a more realistic heterogeneous-agent economy.

Formally, the following questions are left unanswered in the literature: What are the impacts of loss-averse investors on asset prices in an economy with arbitrageurs, whose preferences do not exhibit loss aversion? Do loss-averse investors maintain a significantly large share of the whole economy wealth in the long run, so that their pricing impacts persist? Without a formal model, it is hard to give a definite answer to both questions. This paper fills this gap by providing such a model.

I develop a heterogeneous-agent model with two (classes of) investors and two tradable assets (a risk-free bond and a risky stock). Both investors have recursive preferences. The first investor, labeled *EZ-investor*, has Epstein-Zin preferences (Epstein and Zin, 1989) and she represents rational investors or arbitrageurs. The second investor is called *LA-investor*, and he has a recursive preference representation proposed by Barberis and Huang (2007, 2009).¹ The LA-investor departs from the EZ-investor in the way in which he evaluates his investment in the stock market: he derives utility from investing in the stock both indirectly, via its contribution to his lifetime consumption, and directly, via its resulting fluctuations in his financial wealth, and he is more sensitive to losses than to gains (loss aversion).

¹Throughout this paper, I will use she/her to refer to the EZ-investor and use he/him to refer to the LA-investor.

I choose Epstein-Zin preferences to represent arbitrageurs partly because they help me to separate out the impact of loss aversion on the LA-investor's wealth dynamics (survival) and asset prices, as under Barberis and Huang's (2007, 2009) representation, the difference between the LA-investor's preference and Epstein-Zin preferences is summarized by only one parameter. However, there are also two substantive reasons. First, Epstein-Zin preferences allow the separation between the risk-aversion parameter and the elasticity of intertemporal substitution parameter (EIS henceforth). These two parameters presumably have very different roles in determining investors' survival prospects, as the existing market-selection literature suggests that portfolio decisions, which are more related to risk aversion, and saving behaviors, which are more related to EIS, affect survival in different ways. Second, Epstein-Zin preferences deserve more serious investigation on their own, as the recent literature, such as the long-run risk models (Bansal and Yaron, 2004), has shown that Epstein-Zin preferences help to explain many salient features of the financial market.

My findings can be summarized as follows. On the one hand, if investors only differ in whether deriving loss-aversion utility, then the LA-investor will lose out over time and have no impact on asset prices in the long-run for economies with empirically relevant parameter values (see Subsection 1.4). When the EZ-investor is more risk tolerant, this selection mechanism is stronger in the sense that it does not require the EZ-investor to control a large fraction of wealth to significantly attenuate the effect of loss aversion on prices (see Subsection 1.4.2). On the other hand, when multi-dimensional heterogeneity in

preferences is recognized, say, when the LA-investor and the EZ-investor have different EIS parameters or time-patience parameters, the first result can be easily overturned. For instance, in a calibrated economy, a difference in the time-patience factor as small as two percent can justify the long-run dominance of the LA-investor in the financial market. Therefore, by providing these two results, my paper develops a framework to quantify the effect on survival and asset prices of the difference in investors' preferences.

The first result is obtained through the endogenous difference in investors' equilibrium portfolio choices. It is well-known that as an investor's utility approaches the log utility, his/her expected wealth growth rate increases. Under empirically plausible parameter values, the EZ-investor is more risk averse than the log utility, but the nature of loss aversion makes the LA-investor act as if he is more risk averse than the EZ-investor and therefore further from the log utility. Of course, loss aversion also causes the LA-investor's saving behavior to be different from the EZ-investor's. It might be thought, therefore, that this loss-aversion-induced difference in savings might allow the LA-investor to survive. However, my paper demonstrates that this is not the case in a calibrated economy.

The reason is as follows. Whether the LA-investor saves more or less than the EZ-investor depends on the value of EIS. When the common EIS of both investors is greater than one, the substitution effect is the dominant force determining the investor's saving behavior. The presence of loss aversion makes the LA-investor's future prospect less attractive relative to that of the EZ-investor, thereby causing him to save less, which in turn further hurts his

survival prospects. When the common EIS is less than one, the income effect dominates, and because the presence of loss aversion reduces the LA-investor's future prospects, the income effect implies that he consumes less or saves more than the EZ-investor. However, the difference in their savings declines with the wealth share controlled by the EZ-investor. This is because as the EZ-investor controls more wealth, her lower saving rate raises the risk-free rate, which in turn increases the current consumption of the LA-investor, as the risk-less asset is his primary investment vehicle given the kink at his preferences. As a result, when the LA-investor's wealth erodes because of his adverse portfolio decisions, his saving rate decreases as well, which further drags down his wealth accumulation.

The second result --- that the LA-investor can survive if he has a different EIS parameter or time-patience parameter than the EZ-investor --- is obtained through the endogenous difference in investors' saving behaviors. The intuition is straightforward: When the LA-investor has a larger EIS parameter or time-patience parameter, his saving rate is larger than that of the EZ-investor. This favors his long-run survival. The point of my paper is to quantify this effect. For example, in a calibrated economy, when the EZ-investor's EIS takes a value of 0.5, it is sufficient for the LA-investor to have an EIS of 0.7 to dominate the market, as this difference in EIS generates a difference in saving rate of almost two percent. Similarly, the LA-investor's disadvantage for survival induced by his portfolio decisions can be overturned if his time-patience parameter increases by two percent. This result echoes Yan (2008) who shows that in a dynamic model populated with CRRA investors, a slight difference in the patience parameter makes it possible for an investor with

incorrect beliefs to dominate the market, even if his beliefs persistently and substantially deviate from the truth.

This paper contributes to two strands of literature. The first one is the market selection literature, which studies what kind of investors will survive and have a price impact in a dynamic economy populated by different types of investors. The basic idea is an application of natural selection to financial markets. So far, this literature has primarily focused on selection over beliefs and not over preferences.² Although the idea of market selection dates back to the early 1950s (Alchian, 1950; Friedman, 1953), rigorous analysis is only recent. De Long, Shleifer, Summers, and Waldman (1991) are the first who cast doubts on the idea of market selection. They rely on partial equilibrium analysis and show that investors with incorrect beliefs can survive. Blume and Easley (1992) show that incorrect beliefs can be an advantage for survival in models with endogenous asset prices but exogenous savings decisions. Sandroni (2000), Blume and Easley (2006) and Yan (2008) endogenize both savings and portfolio decisions and show that only investors with beliefs closest to the objective probabilities will survive in economies with bounded aggregate endowment or relative risk aversion. Kogan, Ross, Wang, and Westerfield (2009) demonstrate that in economies with unbounded endowment or relative risk aversion, investors with incorrect beliefs may survive.

²One exception is Condie (2008), who studies the market selection problem for an economy populated with ambiguity averse investors and expected-utility investors.

Investors in all of the above models have time-separable utility functions. Borovička (2009) has recently studied the belief-selection problem in an economy with Epstein-Zin preferences and found that agents with distorted beliefs are not driven out of the market for an empirically relevant range of parameters. Other studies on market selection consider issues related to incomplete markets (Coury and Sciubba, 2005; Sandroni, 2005; Blume and Easley, 2006; Gallmeyer and Hollifield, 2008; Cao, 2009), imperfect competition (Palomino, 1996; Kyle and Wang, 1997), and asymmetric information and learning (Mailath and Sandroni, 2003; Sciubba, 2005; Cogley and Sargent, 2009). Instead of studying belief selection, my paper studies preference selection in frictionless and complete-market economies, and it is the first study on the market-selection problem between loss aversion and Epstein-Zin preferences.

The second strand of related literature considers the role of loss aversion in determining trading behavior, asset prices and trading volumes. Loss aversion is a key feature of prospect theory proposed by Kahneman and Tversky (1979) and means that investors are more sensitive to reductions in the value of their financial wealth than to gains. Berkelaar, Kouwenberg and Post (2004), Gomes (2005) and Kyle, Ouyang and Xiong (2006) study the optimal portfolio choice problem under loss aversion. Benartzi and Thaler (1995) were the first to use loss aversion to explain the equity premium puzzle. Barberis, Huang and Santos (2001) extend Benartzi and Thaler's setting to a dynamic model and find that combining loss aversion and the house-money effect helps to explain the aggregate stock market. Barberis and Huang (2001) find that loss aversion is also useful in understanding the value effect in the cross-section of

stock returns. Grünea and Semmler (2008) study a production economy and find that a model incorporating loss aversion can match data much better than pure consumption-based asset-pricing models, including the habit formation variant. McQueen and Vorkink (2004) show that loss aversion can generate the asymmetric GARCH properties of stock returns. Barberis and Huang (2007, 2009) propose a preference specification that incorporates both loss aversion and narrow framing and study its applications in portfolio choice and asset pricing.

All of the above-mentioned asset-pricing models are conducted in a representative-agent framework. Gomes (2005) and Berkelaar and Kouwenberg (2009) explore the interaction between loss-averse investors and expected utility maximizers; here, the former study focuses more on the implications of loss aversion for trading volumes, while the later studies asset prices and volatility. However, both studies have a finite horizon model and are therefore unable to answer the question of whether loss-averse investors survive and affect prices in the long run. The model proposed in the present paper is suitable for analyzing survival and pricing impact and helps to understand under what conditions and to what extent the results obtained in the asset pricing models incorporating loss aversion are valid.

The remainder of this paper is organized as follows. Subsection 1.2 outlines the model, and Subsection 1.3 characterizes the equilibrium. Subsection 1.4 discusses the implications for survival and price impact of loss aversion when it is the only difference in investors' preferences. Subsection 1.5 discusses its implications for survival when investors have different EIS parameters or time-

patience parameters. Subsection 1.6 concludes.

1.2 The Model

Consider a pure exchange economy with one consumption good, which is the numeraire. Time is discrete and lasts forever: $t = 0, 1, 2, \dots$. There are two assets --- a risk-free bond and a risky stock --- traded competitively in the market. The bond is in zero net supply and earns a gross interest rate of $R_{f,t}$ between time t and $t+1$. The stock is in limited supply (normalized as 1) and it represents a claim to a stream of consumption good represented by the dividend sequence $\{D_t\}_{t=0}^{\infty}$. The stock is traded in a competitive market at price P_t . Let $R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$ and $f_t = \frac{P_t}{D_t}$ be the gross return on the stock between time t and $t+1$ and the price-dividend ratio at time t , respectively.

The dividend growth rate $\theta_{t+1} = \frac{D_{t+1}}{D_t}$ is i.i.d. over time and follows a distribution given by

$$\theta_{t+1} = \begin{cases} \theta_H & \text{with probability } \pi_H \\ \theta_L & \text{with probability } \pi_L \end{cases}, \quad (1)$$

with $0 < \theta_L < \theta_H$, $0 < \pi_H < 1$ and $\pi_L = 1 - \pi_H$. I intentionally choose a binomial distribution of the dividend growth rate process so that the two tradable assets deliver a dynamically complete financial market. By doing so, I ensure that my results on survival are driven by the difference in investors' preferences and not by the assumed financial-market structure. This concern is important because whether the market-selection argument is valid depends crucially on the completeness of financial markets (see, among others, Blume and Easley, 2006; Cao, 2009).

Note that I have followed the literature in assuming that the aggregate consumption and aggregate dividends are equal.³ Under this assumption, even a representative-agent economy with loss-aversion preferences cannot match the historical equity premium,⁴ as the equilibrium stock returns are not volatile enough to scare the loss-averse investor of holding the stock. I have conducted an analysis to extend the baseline model to a three-asset setting which generates the historical equity premium via a combination of loss aversion and narrow framing, and have found that all my main results hold in this extended model. To save space, such an analysis is not reported in this paper.

The economy is populated by two (classes of) investors, who are distinguished by their preferences. The first investor, labeled EZ-investor, derives utility from intertemporal consumption plans according to Epstein-Zin preference specifications (Epstein and Zin, 1989). The second investor, labeled LA-investor, is the investor emphasized in the behavioral finance literature, such as Benartzi and Thaler (1995), Barberis, Huang and Santos (2001), Barberis, Huang and Thaler (2006) and Barberis and Huang (2007, 2009). This investor gets utility not only from consumption but also from fluctuations in the value of his stock holdings, and he is loss-averse over these fluctuations.

³For consumption-based models, see, among others, Lucas (1978) and Mehra and Prescott (1985); for models studying loss aversion, see, among others, Gomes (2005) and Berkelaar and Kouwenberg (2009).

⁴See the first economy studied by Barberis, Huang and Santos, 2001, and Subsection 1.4.1 below.

I use the preference specification developed by Barberis and Huang (2007, 2009) to describe the LA-investor's preferences. According to this specification, the EZ-investor's preference is simply a degenerate case of the LA-investor's preference, where the parameter controlling the term related to loss aversion is set to be zero. I therefore write down a uniform preference formulation for both investors as follows.

Formally, the time t utility of investor i (=EZ, LA) is given by

$$U_{i,t} = H_i \left[C_{i,t}, \mu_i(U_{i,t+1} | I_t) + b_i E_t[v(G_{i,t+1})] \right] \quad (2)$$

where $b_{EZ} = 0$ and $b_{LA} \geq 0$. Here, $H_i(\cdot, \cdot)$ is the aggregator function, which combines current consumption $C_{i,t}$ and the certainty equivalent of future utility to generate current utility $U_{i,t}$. It takes the form

$$H_i(C, X) = \begin{cases} [(1-\beta_i)C^{\rho_i} + \beta_i X^{\rho_i}]^{1/\rho_i}, & \text{if } 0 \neq \rho_i < 1, \\ C^{1-\beta_i} X^{\beta_i}, & \text{if } \rho_i = 0, \end{cases} \quad (3)$$

where $0 < \beta_i < 1$ is investor i 's time patience factor. Parameter ρ_i determines the investor's elasticity of intertemporal substitution (EIS): $EIS_i = 1/(1 - \rho_i)$.

Function $\mu_i(U_{i,t+1} | I_t)$ is the certainty equivalent of the random future utility $U_{i,t+1}$ conditional on time t information I_t , and it has the form

$$\mu_i(U | I_t) = \begin{cases} [E_t(U^{\zeta_i})]^{1/\zeta_i}, & \text{if } 0 \neq \zeta_i < 1, \\ \exp[E_t(\log(U))], & \text{if } \zeta_i = 0, \end{cases} \quad (4)$$

where $E_t(\cdot) \equiv E(\cdot | I_t)$ is the expectation operator conditional on information I_t , and where parameter ζ_i determines the investor's risk attitude toward aggregate future utility, as the implied parameter $RA_i = 1 - \zeta_i$ is the investor's relative risk aversion coefficient. Both investors entertain the same belief, which coincides with the objective probability.

Up to this point, the investor's preference is entirely standard. What is non-standard is that a new term, $b_i E_t[v(G_{i,t+1})]$, is added to the second argument of $H_i(\cdot, \cdot)$, allowing the investor to get utility directly from investing in the stock. This term captures the non-consumption utility that the agent derives directly from the specific gamble he is facing by investing in the stock rather than just indirectly via this gamble's contribution to the next period's wealth and the resulting consumption, since the latter has already been captured by the certainty equivalent function, $\mu_i(U_{i,t+1} | I_t)$. To ease exposition, I refer to this new term as loss aversion utility, and its components --- parameter b_i , argument $G_{i,t+1}$, and function $v(\cdot)$ --- are further specified as follows.

First, parameter b_i determines the relative importance of the loss-aversion utility term in the investor's preference. For the EZ-investor, $b_{EZ} = 0$, meaning that she derives no direct utility from financial wealth fluctuations. For the LA-investor, $b_{LA} > 0$, meaning that, to a certain extent, his utility function depends on the outcome of his stock investment over and above what that outcome implies for total wealth risk.

Second, variable $G_{i,t+1}$ defines the gamble that investor i is taking by investing in the stock. Specifically, let $W_{i,t}$ be investor i 's wealth at the beginning of time

t , and let $s_{i,t}$ be the fraction of post-consumption wealth allocated to the stock. Then this investment portfolio puts the investor to a position of taking a gamble represented by

$$G_{i,t+1} = s_{i,t} (W_{i,t} - C_{i,t}) (R_{t+1} - R_{f,t}), \quad (5)$$

that is, the amount invested in the stock, $s_{i,t} (W_{i,t} - C_{i,t})$, multiplied by its return in excess of the risk-free rate, $R_{t+1} - R_{f,t}$. As is standard in the literature (e.g., Barberis and Huang, 2001, 2007, 2009; Gomes, 2005; Barberis and Xiong, 2009), here the risk-free rate, $R_{f,t}$, is assumed to be the reference point determining whether a particular outcome is treated as a gain or a loss: as long as $s_{i,t} > 0$, the stock's return is only counted as a gain (loss) if it is larger (smaller) than the risk-free rate.

Finally, function $v(\cdot)$ determines how the investor evaluates the gains and losses. I follow Barberis and Huang (2007, 2009) in assuming a piecewise-linear function of $v(\cdot)$:

$$v(G) = \begin{cases} G, & \text{if } G \geq 0, \\ \lambda G, & \text{if } G < 0, \end{cases} \quad (6)$$

with $\lambda > 1$. This function assigns positive utility to gains and negative utility to losses. More importantly, it assigns greater negative utility to losses than positive utilities to gains of the same magnitude. This feature is known as loss aversion in the literature, and it is the behavioral bias that the LA-investor exhibits. Parameter λ controls the degree of loss aversion. Specifically, a one-dollar loss brings the investor $\lambda > 1$ units of negative non-consumption utility, while a one-dollar gain brings him only one unit of positive non-

consumption utility.

As Barberis and Huang (2007, 2009) point out, the preference specification in equations (2)-(6), along many dimensions, improves upon another popularly adopted specification which simply attaches a loss-aversion term to the period felicity function in the expected-utility framework (e.g., Barberis and Huang, 2001; Barberis, Huang and Santos, 2001; McQueen and Vorkink, 2005; Grüne and Semmler, 2008). First, it admits an explicit value function, making it easy to evaluate whether the preference parameters are reasonable. Second, it offers an advantage particular to the current paper: the LA-investor's preference nests the EZ-investor's preference, and as a result, I can easily isolate the impact of loss aversion by simply adjusting the parameters b_i and λ .

To summarize, the economy is characterized by the following two group of exogenous parameters: (i) technology parameters: θ_H , θ_L , π_H and π_L ; and (ii) preference parameters: b_{LA} , λ , $\{\beta_i, \rho_i, \zeta_i\}_{i=EZ, LA}$. The technology is defined by equation (1), and the preferences are defined by equations (2)-(6).

1.3 Equilibrium

I consider Markov equilibria in which price-dividend ratios, the risk-free rate, and the optimal consumption and portfolio decisions are all functions of a state variable and in which the state variable evolves according to a Markov process. The Markov state variable ω_t is the LA-investor's wealth as a fraction of aggregate wealth:

$$\omega_t = \frac{W_{LA,t}}{W_{LA,t} + W_{EZ,t}}. \quad (7)$$

Intuitively, ω_t captures the state of the economy, as it determines the strength of the pricing impact of the LA-investor's trading behavior.

Formally, a Markov equilibrium consists of (i) a stationary price-dividend ratio function, $f : [0,1] \rightarrow \mathbb{R}_{++}$, (ii) a risk-free rate function, $R_f : [0,1] \rightarrow \mathbb{R}_{++}$, (iii) a pair of consumption propensity functions,⁵ $\alpha_{LA} : [0,1] \rightarrow [0,1]$ and $\alpha_{EZ} : [0,1] \rightarrow [0,1]$, (iv) a pair of stock investment policies, $s_{LA} : [0,1] \rightarrow \mathbb{R}$ and $s_{EZ} : [0,1] \rightarrow \mathbb{R}$, and (v) a transition function of the state variable, $\omega : [0,1] \times \{\theta_H, \theta_L\} \rightarrow [0,1]$, such that (i) the consumption policy functions and the portfolio policy functions maximize investors' preferences given the distribution of the equilibrium return processes; (ii) goods and securities markets clear; and (iii) the transition function of the state variable is generated by investors' optimal decisions and the exogenous dividend-growth rate process (i.e., equation [dividend]). I next go through investors' decision problems and the market clearing conditions to construct such an equilibrium.

1.3.1 Investors' Decisions

It may be helpful to summarize the investors' problem here. Investor i chooses consumption $C_{i,t}$ and the fraction of post-consumption wealth allocated to the stock $s_{i,t}$ to maximize

$$U_{i,t} = H_i \left[C_{i,t}, \mu_i(U_{i,t+1} | I_t) + b_i E_t [v(G_{i,t+1})] \right]$$

⁵Consumption propensity is the ratio of consumption over wealth.

subject to the definition of capital gains/losses in stock investment

$$G_{i,t+1} = s_{i,t} (W_{i,t} - C_{i,t}) (R_{t+1} - R_{f,t})$$

and to the standard budget constraint

$$W_{i,t+1} = (W_{i,t} - C_{i,t}) M_{i,t+1},$$

where

$$M_{i,t+1} = R_{f,t} + s_{i,t} (R_{t+1} - R_{f,t}) \quad (8)$$

is the gross return on the investor's portfolio, and functions $H_i(\cdot, \cdot)$, $\mu_i(\cdot)$, and $v(\cdot)$ are given by equations (3), (4), and (6), respectively.

For brevity, I only derive the first-order conditions characterizing the investor's optimal decisions for the case of non-unit elasticity of intertemporal substitution (i.e., for the case of $EIS_i \neq 1$, or of $\rho_i \neq 0$ in the aggregator function $H_i(\cdot, \cdot)$). The first-order conditions for the case of a unit EIS can be derived similarly.

The Bellman equation of the investor's problem is

$$\begin{aligned} U_{i,t} &\equiv J_i(W_{i,t}, I_t) \\ &= \max_{C_{i,t}, s_{i,t}} \left[(1 - \beta_i) C_{i,t}^{\rho_i} + \beta_i \left[\mu \left(J(W_{i,t+1}, I_{t+1}) \mid I_t \right) + b_i E_t (v(G_{i,t+1})) \right]^{\rho_i} \right]^{1/\rho_i}. \end{aligned}$$

Because functions $H_i(\cdot, \cdot)$, $\mu_i(\cdot)$, and $v(\cdot)$ are all homogeneous of degree one, the indirect value function $J_i(W_{i,t}, I_t)$ is also homogeneous of degree one:

$$J_i(W_{i,t}, I_t) = A_i(I_t)W_{i,t} \equiv A_{i,t}W_{i,t}.$$

Therefore,

$$A_{i,t}W_{i,t} = \max_{C_{i,t}, s_{i,t}} \left[(1 - \beta_i)C_{i,t}^{\rho_i} + \beta_i(W_{i,t} - C_{i,t})^{\rho_i} \left[\frac{\mu(A_{i,t+1}M_{i,t+1} | I_t)}{+ b_i E_t[v(s_{i,t}(R_{t+1} - R_{f,t}))]} \right]^{\rho_i} \right]^{1/\rho_i},$$

which implies that the consumption and portfolio decisions are separable.

In particular, the portfolio decision is determined by

$$B_{i,t}^* = \max_{s_{i,t}} [\mu(A_{i,t+1}M_{i,t+1} | I_t) + b_i E_t[v(s_{i,t}(R_{t+1} - R_{f,t}))]] \quad (9)$$

and after defining the consumption propensity as

$$\alpha_{i,t} = C_{i,t} / W_{i,t},$$

the consumption decision is made based on

$$A_{i,t} = \max_{\alpha_{i,t}} [(1 - \beta_i)\alpha_{i,t}^{\rho_i} + \beta_i(1 - \alpha_{i,t})^{\rho_i} (B_{i,t}^*)^{\rho_i}]^{1/\rho_i}. \quad (10)$$

The first-order condition for optimal consumption propensity $\alpha_{i,t}^*$ is⁶

$$B_{i,t}^* = \left(\frac{1 - \beta_i}{\beta_i} \right)^{1/\rho_i} \left(\frac{\alpha_{i,t}^*}{1 - \alpha_{i,t}^*} \right)^{1-1/\rho_i}. \quad (11)$$

Combining equations (10) and (11) delivers

$$A_{i,t} = (1 - \beta_i)^{1/\rho_i} (\alpha_{i,t}^*)^{1-1/\rho_i},$$

which, by the recursive structure, in turn implies

⁶All of the first-order conditions of the investor's problem are both necessary and sufficient, as the objective functions are concave.

$$A_{i,t+1} = (1 - \beta_i)^{1/\rho_i} (\alpha_{i,t+1}^*)^{1-1/\rho_i}. \quad (12)$$

Substituting equations (11) and (12) into equation (9) gives the following single program, which summarizes the investor's consumption and portfolio decisions:

$$\left(\frac{1 - \beta_i}{\beta_i} \right)^{1/\rho_i} \left(\frac{\alpha_{i,t}^*}{1 - \alpha_{i,t}^*} \right)^{1-1/\rho_i} = \max_{s_{i,t}} \left\{ \mu \left[(1 - \beta_i)^{1/\rho_i} (\alpha_{i,t+1}^*)^{1-1/\rho_i} M_{i,t+1} \mid I_t \right] + b_i E_t [v(s_{i,t}(R_{t+1} - R_{f,t}))] \right\}. \quad (13)$$

As a consequence, solving the investor's partial-equilibrium problem boils down to solving a fixed-point problem defined by the first-order condition and the value function of the above maximization problem. Specifically, in the Markov equilibrium, the investor's consumption policy and investment policy are both functions of the state variable ω_t : $s_{i,t}^* = s_i(\omega_t)$, $\alpha_{i,t}^* = \alpha_i(\omega_t)$. The first-order condition and the value function of program (13) thus form a system of two equations with these two unknown functions $s_i(\cdot)$ and $\alpha_i(\cdot)$. Given the equilibrium asset return processes (R_{t+1} and $R_{f,t}$), these partial equilibrium optimal policies can be computed.

It needs certain carefulness to derive the first-order conditions for the portfolio choice, as the utility function, $v(\cdot)$, the function that the investor uses to evaluate gains/losses, is not differentiable everywhere but instead has a kink at the origin. As will become clear in the subsequent analysis, it is this non-differentiability at the origin that is responsible for the non-participation of the LA-investor in the stock market. Formally, the investment optimality is characterized by the following conditions:⁷

⁷To be precise, the conditions apply to the case of non-unit risk aversion, i.e.,

$$\begin{aligned}
& FOC_{i,+} \\
& \equiv (1 - \beta_i)^{1/\rho_i} [E_t(\alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i} M_{i,t+1}^{\zeta_i})]^{1/\zeta_i-1} E_t[\alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i} M_{i,t+1}^{\zeta_i-1} (R_{t+1} - R_{f,t})] \\
& \quad + b_i E_t[v(R_{t+1} - R_{f,t})] \\
& = 0, \text{ for } s_{i,t}^* > 0,
\end{aligned} \tag{14}$$

$$\begin{aligned}
& FOC_{i,-} \\
& \equiv (1 - \beta_i)^{1/\rho_i} [E_t(\alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i} M_{i,t+1}^{\zeta_i})]^{1/\zeta_i-1} E_t[\alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i} M_{i,t+1}^{\zeta_i-1} (R_{t+1} - R_{f,t})] \\
& \quad - b_i E_t[v(R_{f,t} - R_{t+1})] \\
& = 0, \text{ for } s_{i,t}^* < 0,
\end{aligned} \tag{15}$$

$$FOC_{i,+} \leq 0 \text{ and } FOC_{i,-} \geq 0, \text{ for } s_{i,t}^* = 0. \tag{16}$$

In particular, as for the EZ-investor, the expressions of $FOC_{i,+}$ and $FOC_{i,-}$ are the same because $b_{EZ} = 0$. Therefore, her first-order conditions are independent of the sign of her optimal stock investment $s_{EZ,t}^*$:

$$E_t[\alpha_{EZ,t+1}^{(1-1/\rho_{EZ})\zeta_{EZ}} M_{EZ,t+1}^{\zeta_{EZ}-1} (R_{t+1} - R_{f,t})] = 0. \tag{17}$$

This also makes sense: the non-differentiability of preferences comes from the non-differentiability of the loss aversion utility. Since the EZ-investor does not derive any loss aversion utility at all, her utility function is differentiable everywhere, and the first-order conditions are thus the same for all optimal stock investments.

they are true when $RA_i \neq 1$ or $\zeta_i \neq 0$ in the certainty-equivalent function $\mu(\cdot)$. As for the case of a unit risk aversion, simply replace the first terms with $(1 - \beta_i)^{1/\rho_i} e^{(1-1/\rho_i)E_t \log(\alpha_{i,t+1}^*) + E_t \log(M_{i,t+1})} E_t\left(\frac{R_{t+1} - R_{f,t}}{M_{i,t+1}}\right)$, which can be obtained from the limiting formula, $\lim_{\zeta_i \rightarrow 0} [E_t(x^{\zeta_i})]^{1/\zeta_i} = e^{E_t[\log(x)]}$.

1.3.2 Stock Prices and Wealth Dynamics

In this subsection, I rely on market-clearing conditions to derive the expression of price-dividend ratios $f_t \equiv P_t / D_t$ and the evolution of the state variable ω_t . In the Markov equilibrium, the price-dividend ratio f_t is a function of the state variable ω_t :

$$f_t = f(\omega_t).$$

The good market-clearing condition is

$$C_{EZ,t} + C_{LA,t} = D_t. \quad (18)$$

Using the definition of consumption propensity, I can express the consumption levels as products of consumption propensity functions and individual wealth levels:

$$C_{EZ,t} = \alpha_{EZ}(\omega_t)W_{EZ,t} \text{ and } C_{LA,t} = \alpha_{LA}(\omega_t)W_{LA,t}.$$

Then, substituting the above expressions into the good-market clearing condition gives

$$\alpha_{EZ}(\omega_t)W_{EZ,t} + \alpha_{LA}(\omega_t)W_{LA,t} = D_t. \quad (19)$$

Let $W_t = W_{EZ,t} + W_{LA,t}$ be the aggregate wealth of the whole economy at time t .

Recall that the definition of ω_t in equation (7) implies that $W_{EZ,t} = (1 - \omega_t)W_t$ and

$W_{LA,t} = \omega_t W_t$. Therefore, equation (19) becomes

$$[\alpha_{EZ}(\omega_t)(1 - \omega_t) + \alpha_{LA}(\omega_t)\omega_t]W_t = D_t,$$

which implies

$$W_t = \frac{D_t}{\alpha_{EZ}(\omega_t)(1 - \omega_t) + \alpha_{LA}(\omega_t)\omega_t}. \quad (20)$$

Because the bond is zero net supply, and the stock has a net supply of one share, the aggregate economy wealth is also equal to the stock price plus its

dividend:

$$W_t = P_t + D_t. \quad (21)$$

Combining equations (20) and (21) gives the price-dividend ratio function:

$$f(\omega_t) = \frac{(1-\omega_t)\alpha_{EZ}(\omega_t)}{(1-\omega_t)\alpha_{EZ}(\omega_t) + \omega_t\alpha_{LA}(\omega_t)} \frac{1-\alpha_{EZ}(\omega_t)}{\alpha_{EZ}(\omega_t)} + \frac{\omega_t\alpha_{LA}(\omega_t)}{(1-\omega_t)\alpha_{EZ}(\omega_t) + \omega_t\alpha_{LA}(\omega_t)} \frac{1-\alpha_{LA}(\omega_t)}{\alpha_{LA}(\omega_t)}. \quad (22)$$

Equation (22) says that the price-dividend ratios in the heterogeneous agent economy are equal to a weighted average of two terms: $\frac{1-\alpha_{EZ}(\omega_t)}{\alpha_{EZ}(\omega_t)}$ and $\frac{1-\alpha_{LA}(\omega_t)}{\alpha_{LA}(\omega_t)}$. In fact, the expressions of these two terms correspond to the price-dividend ratios in the representative-agent economies populated only by the EZ-Investor and by the LA-investor, respectively.⁸ So, roughly speaking, the price-dividend ratios in a heterogeneous economy is the weighted average of the price-dividend ratios in representative-agent economies, although the weight is not simply the wealth share but is instead a rather complicated expression related to the wealth share and investors' optimal consumption policies.

Given the price-dividend ratio function $f_t = f(\omega_t)$ and the Markov structure of the state variable evolution $\omega_{t+1} = \omega(\omega_t, \theta_{t+1})$, the distribution of stock returns R_{t+1} also has a Markov structure and is determined by

$$R(\omega_t, \theta_{t+1}) \equiv R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{P_{t+1}/D_{t+1} + 1}{P_t/D_t} \frac{D_{t+1}}{D_t} = \frac{f(\omega(\omega_t, \theta_{t+1})) + 1}{f(\omega_t)} \theta_{t+1}. \quad (23)$$

⁸To see this, note that, in a representative agent economy, the agent holds the whole share of the stock and consumes the entire dividend, which means that $\alpha_{i,t} W_{i,t} = \alpha_{i,t} (P_t + D_t) = D_t$ and thus $P_t/D_t = (1 - \alpha_{i,t})/\alpha_{i,t}$.

I now turn to examine how the state variable, ω_t , evolves over time. The gross return to the LA-investor's optimal portfolio is

$$\begin{aligned} M_{LA}(\omega_t, \theta_{t+1}) &\equiv M_{LA,t+1} = R_{f,t} + s_{LA,t}^* (R_{t+1} - R_{f,t}) \\ &= R_f(\omega_t) + s_{LA}(\omega_t) [R(\omega_t, \theta_{t+1}) - R_f(\omega_t)] \end{aligned} \quad (24)$$

Therefore, the LA-investor's next period wealth is

$$\begin{aligned} W_{LA,t+1} &= [1 - \alpha_{LA}(\omega_t)] W_{LA,t} M_{LA}(\omega_t, \theta_{t+1}) \\ &= \frac{[1 - \alpha_{LA}(\omega_t)] \omega_t M_{LA}(\omega_t, \theta_{t+1})}{\alpha_{EZ}(\omega_t)(1 - \omega_t) + \alpha_{LA}(\omega_t)\omega_t} D_t, \end{aligned} \quad (25)$$

where the second equation follows from $W_{LA,t} = \omega_t W_t$ and equation (20).

Applying equation (20) one period forward gives

$$W_{t+1} = \frac{D_{t+1}}{\alpha_{EZ}(\omega_{t+1})(1 - \omega_{t+1}) + \alpha_{LA}(\omega_{t+1})\omega_{t+1}}. \quad (26)$$

Combining equations (25) and (26) and recalling the definition of $\omega_{t+1} = \frac{W_{LA,t+1}}{W_{t+1}}$

and $\theta_{t+1} = \frac{D_{t+1}}{D_t}$, I have

$$\omega_{t+1} = \frac{[1 - \alpha_{LA}(\omega_t)] \omega_t M_{LA}(\omega_t, \theta_{t+1}) [\alpha_{EZ}(\omega_{t+1})(1 - \omega_{t+1}) + \alpha_{LA}(\omega_{t+1})\omega_{t+1}]}{[\alpha_{EZ}(\omega_t)(1 - \omega_t) + \alpha_{LA}(\omega_t)\omega_t] \theta_{t+1}} \quad (27)$$

which implicitly determines the evolution of ω_t : $\omega_{t+1} = \omega(\omega_t, \theta_{t+1})$.

Finally, substituting $W_{EZ,t} = (1 - \omega_t)W_t$, $W_{LA,t} = \omega_t W_t$ and equation (20) into the stock-market clearing condition,

$$P_t = s_{EZ,t}^* (1 - \alpha_{EZ,t}^*) W_{EZ,t} + s_{LA,t}^* (1 - \alpha_{LA,t}^*) W_{LA,t},$$

I link investors' policy functions to the price-dividend ratio function as follows:

$$f(\omega_t) = \frac{s_{EZ,t}^* (1 - \alpha_{EZ,t}^*) (1 - \omega_t) + s_{LA,t}^* (1 - \alpha_{LA,t}^*) \omega_t}{\alpha_{EZ}(\omega_t) (1 - \omega_t) + \alpha_{LA}(\omega_t) \omega_t}. \quad (28)$$

To summarize, computing the equilibrium is involved with solving the seven unknown functions, $f(\cdot)$, $R_f(\cdot)$, $\alpha_{LA}(\cdot)$, $\alpha_{EZ}(\cdot)$, $s_{LA}(\cdot)$, $s_{EZ}(\cdot)$, and $\omega(\cdot, \cdot)$ from the system formed by equations (13), (14)-(16), (22)-(24), (27) and (28).

Two remarks are in order. First, although the market is complete in the present project, the standard Pareto efficiency technique commonly used in the market-selection literature (e.g., Blume and Easley, 2006; Yan, 2008; Borovička, 2009; Kogan, Ross, Wang and Westerfield, 2009) cannot be applied here, as the LA-investor's preference depends not only on the intertemporal consumption plans but also on the endogenous stock return process *per se*, thereby making it necessary to explicitly solve the equilibrium. I therefore develop an algorithm based on Kubler and Schmedders (2003) to compute the Markov equilibrium and use simulations to analyze the survival and price impact of the LA-investor. The details of the algorithm are delegated to the appendix.

Second, my analysis ignores the issue of the existence and uniqueness of the equilibrium. As is well-known in the literature, it is hard to establish the general results on the existence and uniqueness of the equilibria in heterogeneous-agent models. Therefore, in the present paper, I simply start my analysis under the assumption that an equilibrium exists and use numerical methods to find this equilibrium. Rigorously speaking, a numerical method can never find the exact equilibrium; what it finds, if anything, is the ε -equilibrium defined by

Kubler and Schmedders (2003), who interpret the computed ε -equilibrium as an approximate equilibrium of some other economy with endowments and preferences that are close to those in the original economy.

1.4 Implications of Loss Aversion for Survival and Price Impacts

In this section, I first analyze the representative-agent economies, that is, economies populated by homogeneous investors (see Subsection 1.4.1). This analysis serves two purposes. First, it verifies the result that loss aversion raises equity premiums, which is well-known in the literature (Benartzi and Thaler, 1995; Barberis, Huang and Santos, 2001). Second, it provides a very useful springboard for my analysis of the heterogeneous-agent economies, as it helps to develop the intuition for how loss aversion changes an investor's investment and saving behaviors. I then move to the more realistic economies populated by both the EZ-investor and the LA-investor and use the algorithm in Appendix 1 to numerically compute the equilibrium price functions, $f(\cdot)$, $R_f(\cdot)$, policy functions, $\alpha_{LA}(\cdot)$, $\alpha_{EZ}(\cdot)$, $s_{LA}(\cdot)$, $s_{EZ}(\cdot)$, and the state variable transition function, $\omega(\cdot, \cdot)$. I use simulations to show how loss aversion affects the investor's survival and pricing impact via changed portfolio decisions in Subsection 1.4.2 and via changed saving behaviors in Subsection 1.4.3.

Before solving the models, I need to calibrate the parameter values. Because I am interested in the implications of preferences, I allow the preference parameters to vary over a certain range while fixing the four technology parameters in equation (1) for all computations and simulations. I interpret one period as one year and follow Mehra and Prescott (1985) in setting

$\pi_H = \pi_L = 1/2$ so that the economy is in booms and recessions with equal

probability. Based on the data spanning the 20th century, the historical mean and volatility of the log consumption growth process are 1.84% and 3.79%, respectively (see Barberis and Huang, 2009). To match these two moments, I set $\theta_H = 1.0579$ and $\theta_L = 0.98069$. Table 1.1 summarizes my choice of technology parameters.

Table 1.1: Technology Parameter Values

I take one period to be one year. Following Mehra and Prescott (1985), I assume that the dividend growth rate takes a high value or a low value with equal probability, so that $\pi_H = \pi_L = 0.5$. The dividend growth rate parameters (θ_H and θ_L) are calibrated to match the historical mean (1.84%) and volatility (3.79%) of the log consumption growth rate.

Parameters	π_H	π_L	θ_H	θ_L
Values	0.5	0.5	1.0579	0.98069

1.4.1 Representative-Agent Economy

In this subsection, I assume that the EZ-investor and the LA-investor have identical preferences; that is, $\beta_{EZ} = \beta_{LA} \equiv \beta$, $\rho_{EZ} = \rho_{LA} \equiv \rho$, $\zeta_{EZ} = \zeta_{LA} \equiv \zeta$ and $b_{EZ} = b_{LA} \equiv b$. As a result, the economy is the well-studied representative-agent economy.

In this case, the representative agent has to hold the stock in equilibrium, so that the first-order condition in equation (14) with $M_{i,t+1} = R_{t+1}$ defines the optimality of the investor's investment decision. As mentioned in the discussions after equation (22), the good-market clearing condition links the

price-dividend ratios f_t to the optimal consumption policy α_t as follows:

$$D_t = (1 - \alpha_t)(P_t + D_t) \Rightarrow f_t = \frac{1 - \alpha_t}{\alpha_t}. \quad (29)$$

Therefore, equations (13), (14) and (29) define a system for three unknowns:

f_t , α_t and $R_{f,t}$. Given the *i.i.d.* investment opportunities, I conjecture that

$$(f_t, \alpha_t, R_{f,t}) = (f, \alpha, R_f), \forall t. \quad (30)$$

The problem can be easily solved using any non-linear solver.

Table 1.2 reports the equilibrium equity premiums, risk-free rates and consumption policies for a variety of combinations of preference parameter values. For all combinations, I hold constant the time patience factor β , the loss-aversion parameter λ and the relative risk-aversion coefficient RA : $\beta = 0.98$, $\lambda = 2.25$ and $RA = 1$ (or $\zeta = 0$). These values are standard in the behavioral finance literature, such as Barberis, Huang and Santos (2001) and Barberis and Huang (2009). The choice of β is motivated to match the low level of the risk-free rate, while the choice of λ is based on the estimation of Tversky and Kahneman (1992). When EIS is equal to one (i.e., $\rho = 0$) and there is no loss aversion (i.e., $b = 0$), setting $RA = 1$ or $\zeta = 0$ reduces the investor's preference to an expected log utility, which is an important benchmark case that the market-selection literature has been focusing on (Hakansson, 1971; De Long, Shleifer, Summers, and Waldmann, 1991; Blume and Easley, 1992).

Table 1.2: Asset Prices and Consumption Policies in Representative Agent Economies

Table 1.2 reports the equilibrium equity premiums, risk-free rates and consumption policies, assuming that investors are identical in preferences, so that $\beta_{EZ} = \beta_{LA} \equiv \beta$, $EIS_{EZ} = EIS_{LA} \equiv EIS$ (or $\rho_{EZ} = \rho_{LA} \equiv \rho$), $RA_{EZ} = RA_{LA} \equiv RA$ (or $\zeta_{EZ} = \zeta_{LA} \equiv \zeta$) and $b_{EZ} = b_{LA} \equiv b$. For all combinations, the following three preference parameters are held constant: $\beta = 0.98$, $\lambda = 2.25$ and $RA = 1$ (or $\zeta = 0$). The technology parameters are fixed at the values in Table 1.1. Panels A, B and C correspond to different values of EIS: $EIS = 1$ ($\rho = 0$), $EIS = 0.5$ ($\rho = -1$) and $EIS = 1.5$ ($\rho = 1/3$). Parameter b controls the relative importance of loss aversion utility in the investor's preferences.

	$E(R_{t+1} - R_{f,t})$ (%)	$E(R_{f,t} - 1)$ (%)	α_t (%)
Panel A: $EIS = 1$			
$b = 0$	0.15	3.86	2.00
$b = 0.02$	0.79	3.22	2.00
$b = 0.2$	1.41	2.60	2.00
Panel B: $EIS = 0.5$			
$b = 0$	0.15	5.79	3.79
$b = 0.02$	0.67	4.76	3.31
$b = 0.2$	1.39	3.29	2.63
Panel C: $EIS = 1.5$			
$b = 0$	0.15	3.23	1.40
$b = 0.02$	0.85	2.76	1.62
$b = 0.2$	1.42	2.38	1.80

Panels A, B and C correspond to different values of EIS: $EIS = 1$ ($\rho = 0$), $EIS = 0.5$ ($\rho = -1$) and $EIS = 1.5$ ($\rho = 1/3$). In each panel, parameter b , which controls the relative importance of the loss aversion utility in the investor's preference, is set at three different values: $b = 0$, $b = 0.02$ and $b = 0.2$. When $b = 0$, the investor's preference does not exhibit loss aversion, and this

economy has been well understood in the literature (e.g., Weil, 1989). When $b > 0$, the investor's preference exhibits loss aversion; such an economy is the focus of behavioral finance, such as Benartzi and Thaler (1995), Barberis, Huang and Santos (2001), and Barberis and Huang (2007, 2009). In particular, for both positive values of b in the table, 0.02 and 0.2, the investor's attitudes to independent *large* monetary gambles are sensible in the sense that both parameterizations of the investor's preference satisfy Barberis and Huang's condition L (2007, p 217); moreover, when $b = 0.02$, the investor's attitude to independent *small* monetary gambles is also sensible; that is, the parameterization corresponding to $b = 0.02$ satisfies Barberis and Huang's condition S (2007, p 219).⁹

Three notable patterns show up in Table 1.2. The first pattern regards the equity premium. In all three panels, when $b = 0$, that is, when loss aversion is absent in the investor's preference, the equity premium is quite small (0.15%) relative to its historical value (6%), which is the well-known equity premium puzzle. Once loss aversion is introduced, the equity premiums are raised significantly. Say, when $b = 0.2$, the model can generate an equity premium as high as 1.4%, which is almost ten times the equity premium corresponding to

⁹The literature cares about investors' attitudes to independent monetary gambles, as it was, in part, the difficulty that researchers encountered in reconciling the equity premium with these attitudes that launched the equity premium literature in the first place. Barberis and Huang's (2007) condition L is: (a)n individual with wealth of \$75,000 should not pay a premium higher than \$15,000 to avoid a 50:50 chance of losing \$25,000 or gaining the same amount. Their condition S is: (a)n individual with wealth of \$75,000 should not pay a premium higher than \$40 to avoid a 50:50 chance of losing \$250 or gaining the same amount.

an economy populated by only EZ-investors. The increased equity premiums still fall short of the empirical value, as in my model, the stock is a claim to the *smooth* aggregate consumption process, and, as a result of the constant equilibrium price-dividend ratios (see equation [30]), the stock returns are not volatile enough to cause the loss-averse investor to be scared of holding the stock.¹⁰ As mentioned before, this mismatch between the model-generated equity premium and the historical equity premium does not have any impact on my analysis. What really matters is that the LA-investor is more reluctant to hold the stock than the EZ-investor, which is also an assumption maintained in the behavioral finance studies relying on loss aversion to explain the equity premium puzzle.

The second pattern concerns the risk-free rate. In all three panels, the risk-free rate decreases with b , the parameter determining the relative importance of loss-aversion utility in the investor's preferences. This occurs because as the investor is more concerned about fluctuations in the value of his financial wealth, he is more inclined to allocate wealth to the safe asset to avoid the potential painful losses associated with the risky asset. This suggests that in a heterogeneous-agent economy populated by both the LA-investor and the EZ-investor, the bond is more attractive to the former than to the latter. When parameter b is fixed, the risk-free rate also decreases with the magnitude of

¹⁰Barberis, Huang and Santos (2001) also study the pricing impact of loss aversion in a representative-agent economy with dividends equal to consumption, and they report an equity premium of 1.26% (see the top part of their Table II), which is close to the equity premium generated in my model (1.4%).

EIS: in a growing economy, a higher EIS makes the investor more likely to save, thereby depressing the interest rate.

The third pattern is about the consumption policy. As is well-known in the literature, when EIS is equal to one, the investor's saving ratio is optimally chosen to be equal to the time patience factor, β . Therefore, in Panel A, the optimal consumption propensity is independent of the relative importance of loss-aversion utility in the investor's preferences; that is, the value of α_t is independent of parameter b . However, when EIS is different from 1, α_t varies with b : α_t decreases (increases) with b when EIS is less (greater) than 1 in Panel B (Panel C). As is standard in the portfolio choice problem for recursive preferences, two forces --- the income effect and the substitution effect --- are at play here. The asymmetric treatment of losses from gains in the loss-aversion utility tends to lower the value, measured in utility terms, of the investor's future investment opportunities; that is, a higher b tends to yield a lower B_t^* in equation (9). This lowered B_t^* has two effects on current consumption: it lowers consumption propensity through the income effect but raises consumption propensity through the substitution effect. When EIS is below 1, the income effect dominates, so that α_t decreases with b ; when EIS is above 1, the substitution effect dominates, and the relationship between α_t and b reverses as a result. The different responses of α_t to b in different cases of EIS suggest that how loss aversion affects the LA-investor's survival might depend on whether EIS is greater than or smaller than 1, as the literature suggests that saving behavior is a key determinant on survival. This will be examined in Subsection 1.4.3.

1.4.2 EIS=1: Portfolio Selection

In this subsection, I study the heterogeneous-agent economy and fix EIS at 1, so that both investors optimally choose to have a constant consumption-wealth ratio: $\alpha_{i,t}^* = 1 - \beta_i$, for $i = \text{EZ}, \text{LA}$. I assume that the preferences of both investors are otherwise identical except that the LA-investor derives loss-aversion utility, while the EZ-investor does not. So, except that $b_{\text{LA}} > 0$, $b_{\text{EZ}} = 0$, all other parameters are the same across investors: $\beta_{\text{EZ}} = \beta_{\text{LA}} \equiv \beta$, $\rho_{\text{EZ}} = \rho_{\text{LA}} \equiv \rho$ and $\zeta_{\text{EZ}} = \zeta_{\text{LA}} \equiv \zeta$. The assumption of a common time-patience parameter implies that both investors have the same endogenous saving rate. The focus of this subsection is therefore essentially how loss aversion changes the LA-investor's portfolio decision, which in turn affects asset prices as well as the LA-investor's long-run survival in a complete financial market.

I define the survival, extinction, dominance and price impact of the LA-investor as follows.

Definition. The LA-investor is said to become extinct if

$$\lim_{t \rightarrow \infty} \omega(t) = 0, \text{ a.s.};$$

to survive if extinction does not occur; and to dominate the market if

$$\lim_{t \rightarrow \infty} \omega(t) = 1, \text{ a.s..}$$

The price impact of loss aversion at state ω is

$$\text{priceimpact}(\omega) = \frac{E(R_{t+1} - R_{f,t} \mid \omega_t = \omega) - E(R_{t+1} - R_{f,t} \mid \omega_t = 0)}{E(R_{t+1} - R_{f,t} \mid \omega_t = 1) - E(R_{t+1} - R_{f,t} \mid \omega_t = 0)}. \quad (31)$$

The definition of survival is standard in the market selection literature, such as Yan (2008) and Kogan, Ross, Wang and Westerfield (2009). The definition of price impact is also intuitive. The common term subtracted in the numerator and in the denominator --- $E(R_{t+1} - R_{f,t} \mid \omega_t = 0)$ --- is the equilibrium equity premium in the traditional representative-agent economy, where all investors have Epstein-Zin preferences, which serves as a benchmark level of the equity premium. The first term in the denominator, $E(R_{t+1} - R_{f,t} \mid \omega_t = 1)$, corresponds to the equity premium obtained in a representative-agent economy populated by LA-investors only. The denominator therefore measures an *upper bound* of the additional equity premium relative to the benchmark level that one could obtain by introducing loss aversion into the investor's preferences. The numerator is the achieved extra equity premium when the LA-investor controls ω fraction of the aggregate wealth in the economy. When $\omega = 0$, $priceimpact(\omega) = 0$, and there is no price impact of loss aversion; when $\omega = 1$, $priceimpact(\omega) = 1$, and loss aversion has a price impact as large as in the representative-agent economy with only LA-investors. Since the variable $priceimpact$ depends only on the state variable, it has a Markov process in equilibrium. I call function $priceimpact(\cdot)$ a *price-impact function*.

To illustrate how the LA-investor's wealth share (ω_t) and his pricing impact ($priceimpact_t$) evolve over time, Table 1.3 reports their distributions at times $t = 50$, 100 , and 1000 years when the LA-investor has initial wealth shares of $\omega_0 = 0.1$, 0.5 , and 0.9 and both investors have a relative risk aversion coefficient of 1 (Panel A) or 3 (panel B). The technology parameters are fixed at the values in Table 1.1, and the other preference parameters are $\beta_{EZ} = \beta_{LA} = 0.98$, $\lambda = 2.25$ and $b_{LA} = 0.02$.

Table 1.3: Survival and Price Impacts: $EIS = 1$

Table 1.3 reports the distributions of the LA-investor's wealth shares (ω_t) and his price impacts ($priceimpact_t$) at times $t = 50$, 100 , and 1000 years when the LA-investor has initial wealth shares of $\omega_0 = 0.1$, 0.5 , and 0.9 as well as both investors have a relative risk aversion coefficient of 1 (Panel A) or 3 (panel B). Both investors have a unit EIS: $EIS_{EZ} = EIS_{LA} = 1$. The other preference parameters are $\beta_{EZ} = \beta_{LA} = 0.98$, $\lambda = 2.25$ and $b_{LA} = 0.02$. The technology parameters are fixed at the values in Table 1.1. Each entry in Table 1.3 has three elements, corresponding, respectively, to the 5%, 50%, and 90% percentiles of the distributions of ω_t or $priceimpact_t$. The quantiles are estimated from the 5000 simulated sample paths at time t .

Panel A: $RA_{EZ} = RA_{LA} = 1$ ($\zeta_{EZ} = \zeta_{LA} = 0$)			
5%, 50% and 95% Quantiles of ω_t			
	$\omega_0 = 0.1$	$\omega_0 = 0.5$	$\omega_0 = 0.9$
$t = 50$	(.0632 .0993 .1451)	(.3022 .4523 .6677)	(.5324 .8070 .9666)
$t = 100$	(.0484 .0885 .1720)	(.2296 .4059 .6929)	(.3928 .6682 .9714)
$t = 1000$	(.0066 .0432 .2415)	(.0246 .1282 .4603)	(.0337 .1592 .5202)
5%, 50% and 95% Quantiles of $priceimpact_t$			
	$priceimpact_0 = .0257$	$priceimpact_0 = .2301$	$priceimpact_0 = .9160$
$t = 50$	(.0156 .0255 .0392)	(.0999 .1902 .4590)	(.2617 .8369 .9713)
$t = 100$	(.0118 .0224 .0480)	(.0689 .1575 .5142)	(.1491 .4601 .9754)
$t = 1000$	(.0015 .0104 .0735)	(.0058 .0340 .1964)	(.0081 .0437 .2494)
Panel B: $RA_{EZ} = RA_{LA} = 3$ ($\zeta_{EZ} = \zeta_{LA} = -2$)			
5%, 50% and 95% Quantiles of ω_t			
	$\omega_0 = 0.1$	$\omega_0 = 0.5$	$\omega_0 = 0.9$
$t = 50$	(.0543 .0849 .1242)	(.2556 .3964 .5499)	(.7752 .8613 .9089)
$t = 100$	(.0340 .0654 .1243)	(.1646 .2867 .5138)	(.6159 .7864 .8963)
$t = 1000$	(.0004 .0023 .0155)	(.0014 .0083 .0429)	(.0039 .0204 .0893)
5%, 50% and 95% Quantiles of $priceimpact_t$			
	$priceimpact_0 = .0949$	$priceimpact_0 = .6464$	$priceimpact_0 = .9477$
$t = 50$	(.0496 .0796 .1205)	(.2805 .5195 .6953)	(.8732 .9260 .9526)
$t = 100$	(.0323 .0602 .1206)	(.1655 .3251 .6602)	(.7543 .8804 .9457)
$t = 1000$	(.0003 .0022 .0137)	(.0012 .0073 .0387)	(.0034 .0181 .0839)

Each entry in Table 1.3 has three elements corresponding, respectively, to the 5%, 50%, and 90% percentiles of the distributions of ω_t or $priceimpact_t$. Say, the first entry means that if both investors have a relative risk aversion coefficient of 1 and if the LA-investor has an initial wealth share of 0.1, then after 50 years, the LA-investor's wealth shares will be lower than 0.0632, 0.0993, or 0.1451 with probabilities of 5%, 50%, and 95%, respectively. These quantiles are obtained from simulations. I first use the algorithm described in Appendix 1 to solve the equilibrium price-dividend ratio function $f(\cdot)$, the risk-free rate function $R_f(\cdot)$ and the state transition function $\omega(\cdot, \cdot)$. For any given ω_0 , I then simulate $N = 5000$ economies. For each economy, I simulate a long time series $\{\theta_t\}_{t=1}^{\infty}$ of $T = 1000$ independent draws from the distribution described in equation (1). I then use the solved function $\omega(\cdot, \cdot)$ to calculate the next-period state ω_{t+1} and use functions $f(\cdot)$ and $R_f(\cdot)$ to calculate the conditional equity premium as well as the resulting price impact $priceimpact_t$ along the way. Finally I use the cross-sectional empirical distributions of ω_t and $priceimpact_t$ at a particular time in point t to represent their population distributions. The quantiles of ω_t and $priceimpact_t$ are therefore estimated from the 5,000 simulated sample paths at time t . In the following discussion, I focus on the 50% quantiles or the second element of each entry.

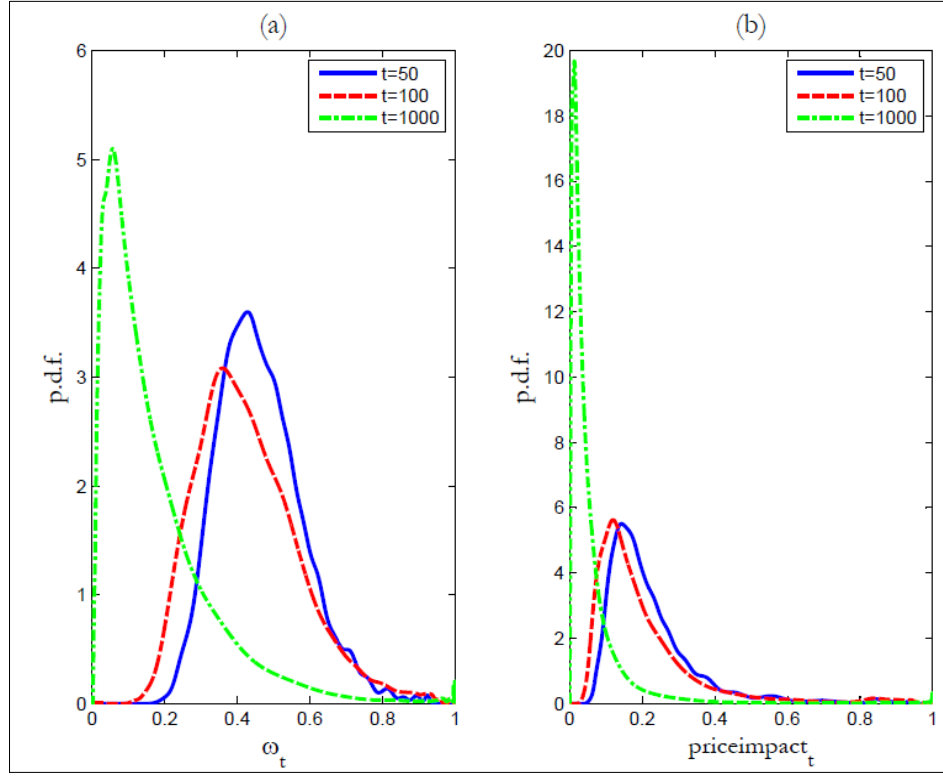


Figure 1.1 Survival and Price Impacts when $EIS_{EZ} = EIS_{LA} = 1$

Figure 1.1 graphs the probability density functions (p.d.f.s) of the LA-investor's wealth shares (ω_t) and his price impacts ($priceimpact_t$) at times $t = 50, 100, 1000$ when both investors have a unit EIS, that is, when $EIS_{EZ} = EIS_{LA} = 1$. The p.d.f.s are estimated non-parametrically from 5000 simulated data. At time 0, each investor has half of the aggregate wealth; that is, $\omega_0 = 0.5$. The preference parameters are $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.98$, $\lambda = 2.25$ and $b_{LA} = 0.02$. The technology parameters are fixed at the values in Table 1.1.

Table 1.3 delivers two important messages. First, both ω_t and $priceimpact_t$

decrease over time. For instance, in Panel A, if each investor initially has half of the total wealth at time 0, then on a typical sample path, which is the sample path that delivers the median of ω_t , the LA-investor's wealth share shrinks to 0.45 over the first 50 years and further to 0.13 over the first 1,000. Along the way, his price impact gradually declines from 0.23 to 0.19 in year 50 and then to 0.03 in year 1000. Figure 1.1 graphically depicts the dynamics of the whole simulated probability density functions (p.d.f.s) of the LA-investor's wealth shares (the left panel) and his price impacts (the right panel) for the case of $\omega_0 = 0.5$ and $RA_{EZ} = RA_{LA} = 1$. The p.d.f.s are estimated nonparametrically from the simulation data in Table 1.3. As time passes, all p.d.f.s shift to the left, illustrating how the LA-investor's wealth shares and thus his impact on asset prices decrease over time with high probability.

The intuition is straightforward. The rate at which investors' wealth grows depends on how close their preferences are to the log utility. In panel A of Table 1.3, the EZ-investor is the log investor, while in Panel B, she is more risk averse than the log investor. As for the LA-investor, loss aversion adds one more layer of risk aversion toward the stock over and above the conventional risk aversion shared by the EZ-investor, which causes him to mimic an investor who is more risk averse, and hence further from log utility, than the EZ-investor. Therefore, the LA-investor vanishes in the long run, and, as a result, so does his price impact.

Basically, the above argument relies on two elements: (i) both investors are at least as risk averse as a log investor; (ii) the LA-investor behaves in a way so that he is more risk averse than the EZ-investor. Both elements are empirically

plausible. On the one hand, the literature suggests that it is unlikely that real investors are less risk averse than log utility because these investors are subject to the St. Petersburg paradox (Samuelson, 1977): they are willing to pay an infinite amount of money for a gamble offering zero with a probability arbitrarily close to one and paying finite amounts in each state of the world. On the other hand, empirical estimates or thought experiments suggest that investors are only mildly risk averse, which is why the equity premium puzzle literature was launched in the first place.¹¹ The behavioral finance literature shows that once loss aversion is incorporated into an investor's preferences, the equity premium increases, suggesting that loss aversion alters the investor's risk attitude in a way that makes him more risk averse than the EZ-investor.

The second message conveyed by Table 1.3 regards the speed of the market selection process. In terms of wealth shares, the process is slow. For example, in all cases, after 50 years, on a typical sample path, the LA-investor loses less than 20% of his initial wealth share; even after 100 years, he still reserves more than half of his initial wealth share.

However, the effectiveness of the market selection mechanism should be judged by how the price impacts and not the wealth shares of the LA-investor

¹¹Kocherlakota (1996, p 52) summarizes that the empirical plausible range of the relative risk aversion coefficient is between 0 and 10 by stating that a vast majority of economists believe that values for [the coefficient of relative risk aversion] above ten (or, for that matter, above five) imply highly implausible behavior on the part of individuals.

change over time, as what one really cares about is whether the behavior of asset prices can largely be captured by models without the LA-investor. Although wealth shares and price impacts are closely related in the long run, their dynamics might be very different in the short run. As a result, the slow declining speed of the LA-investor's wealth share does not mean that the market selection mechanism is not effective in eliminating his pricing impacts. There are at least two forces that break down the link, and both forces are related to the shape of function $priceimpact(\cdot)$, which, as defined by equation (31), associates wealth shares with price impacts. First, it is possible that only a small amount of wealth controlled by the EZ-investor is needed to arbitrage away a large fraction of the LA-investor's impact on asset prices. This will be true when a large part of function $priceimpact(\cdot)$ is flat. Second, it is likely that a small drop in wealth shares leads to a large drop in price impact. This will be true when $priceimpact(\cdot)$ is convex at large wealth shares and concave at small wealth shares.

It turns out the effectiveness of the market selection mechanism in terms of price impacts varies among risk aversions of the EZ-investor. For instance, when each investor has half of the total wealth ($\omega_0 = 0.5$), if the EZ-investor is only mildly risk averse ($RA_{EZ} = 1$), then the LA-investor's price impact is only 0.23 ($priceimpact_0 = 0.23$), but if the EZ-investor has greater risk aversion ($RA_{EZ} = 3$), the LA-investor's price impact rises to 0.65 ($priceimpact_0 = 0.65$). The dynamics of the pricing impact have a similar pattern. Take $\omega_0 = 0.9$, for example. In Panel A of Table 1.3, on a typical sample path, after 100 years, the LA-investor's price impact loses half of its initial value and falls from 0.92 to 0.46. However, in Panel B, when the EZ-investor becomes more risk averse,

the LA-investor's price impact drops by less than 10% for the first 100 years.

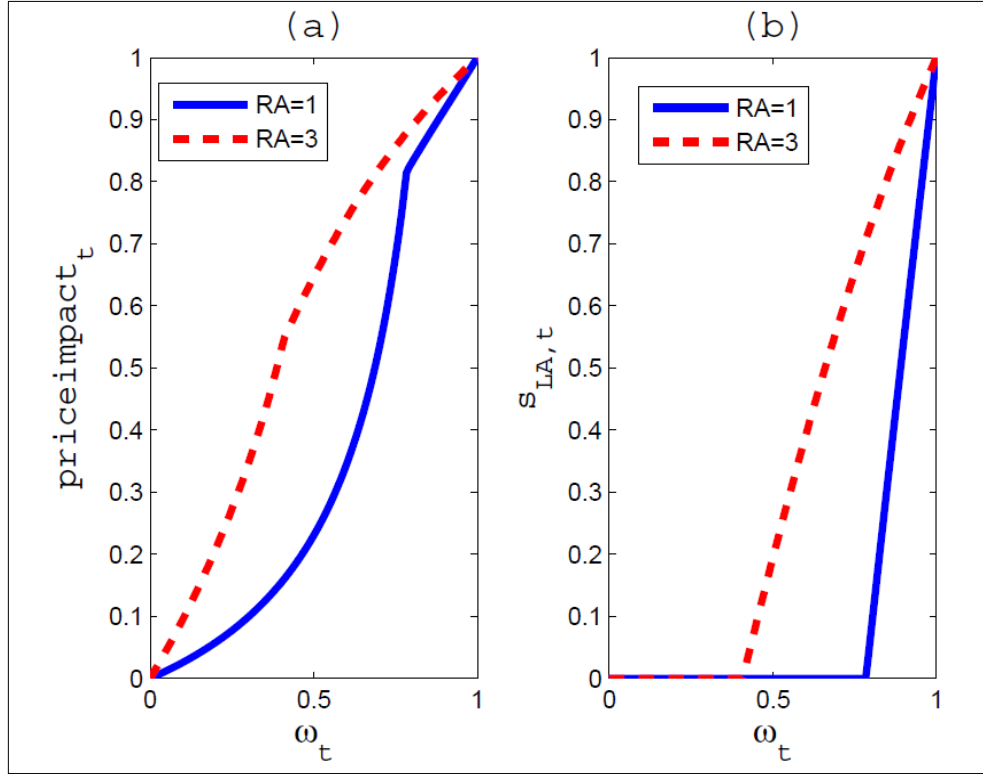


Figure 1.2 Price-Impact Function and Investment Policy when $EIS_{EZ} = EIS_{LA} = 1$

Figure 1.2 graphs the price-impact function as defined by equation (31) the LA-investor's investment policy function $s_{LA}(\cdot)$ when both investors have a unit EIS, that is, when $EIS_{EZ} = EIS_{LA} = 1$. Both investors have a common relative risk aversion coefficient, which can be either 1 (the solid line) or 3 (the dashed line). The other preference parameters are $\beta_{EZ} = \beta_{LA} = 0.98$, $\lambda = 2.25$ and $b_{LA} = 0.02$. The technology parameters are fixed at the values in Table 1.1.

To understand this result, Figure 1.2 displays the function of $priceimpact(\cdot)$ in the left panel and the LA-investor's investment policy function $s_{LA}(\cdot)$ in the right

panel for the same preference and technology parameters as adopted by Table 1.3. Both functions are increasing in ω_t : as the wealth controlled by the LA-investor increases, so do the amount of stock he buys and his price impact. Moreover, both functions, in particular, function $priceimpact(\cdot)$, increase with RA_{EZ} . As a result, for the same level of ω_t , the price impact of the LA-investor is larger when the EZ-investor is more risk averse. This is because a higher risk aversion makes the EZ-investor less aggressive in her trading against the LA-investor.

The price-impact function $priceimpact(\cdot)$ has a kink in the middle. Below the kink, the function is convex, while above the kink, the function is concave. A decline in ω_t leads to the largest drop of $priceimpact_t$ at the kink. Comparing Panel (a) with Panel (b), it can be seen that the location of the kink is determined by the level of the wealth share at which the LA-investor starts to buy the stock. Specifically, because there is a kink in the utility function of the LA-investor, he will allocate nothing to the stock if the expected stock returns are not high enough. If the LA-investor stays out of the stock market, the equity premium is determined by the Euler equations of the EZ-investor, and, as a result, it is close to that found in a representative-agent economy populated only by EZ-investors, which explains the convexity part of the price-impact function. Once the LA-investor starts to buy the stock, his Euler equations start to determine the equity premium and affect the curvature of the price-impact function. Note that the location of the kink approaches 1 as the EZ-investor becomes less risk averse. This means if the LA-investor's price impact is initially very high, large drops of $priceimpact_t$, which occur around the kink, come sooner for a smaller RA_{EZ} .

To summarize, for the unit EIS case, if the LA-investor differs from the EZ-investor only in terms of loss aversion utility, that is, except for $b_{LA} > 0$ and $b_{EZ} = 0$, the other preference parameters are identical across investors, then the LA-investor vanishes in the long run. In addition, the market selection mechanism is more efficient when the EZ-investor is more risk tolerant. In the following analysis, I will focus only on the dynamics of wealth shares, as the relationship between the wealth share dynamics and the pricing impact dynamics obtained in this subsection is largely reserved in the case of a non-unit EIS.

1.4.3 $EIS \neq 1$: Saving Behavior

The analysis in Subsection 1.4.1 suggests that when EIS is not equal to 1, loss aversion can change the investor's saving behavior, which might affect the investor's wealth accumulation and survival prospects. In this subsection, I investigate this possibility in the heterogeneous-agent economy. Again, I assume that the preferences of both investors are identical except that $b_{LA} > 0$ and $b_{EZ} = 0$.

When $EIS_{EZ} = EIS_{LA} > 1$, the intuition in the representative-agent economies implies that the LA-investor consumes more than the EZ-investor, which hurts his survival prospects. Because the previous subsection shows that in the absence of different saving behaviors, the LA-investor already loses to the EZ-investor; then, this extra force coming from saving should cause the LA-investor to vanish at a faster speed. This is indeed the case, as verified by Figure 1.3, which plots the distributions of ω_t in years 50, 100, and 1000, for

the case of $EIS_{EZ} = EIS_{LA} = 1.5$, when each investor has half of the total wealth at time 0. The distributions are obtained in the same way as in Figure 1.1, and the technology parameters are fixed at the values in Table 1.1, while the other preference parameters are $\beta_{EZ} = \beta_{LA} = 0.98$, $RA_{EZ} = RA_{LA} = 1$, $\lambda = 2.25$ and $b_{LA} = 0.02$. Indeed, the p.d.f.s of ω_t shift to the left as time passes, suggesting that the LA-investor is losing his wealth share and price impact over time.

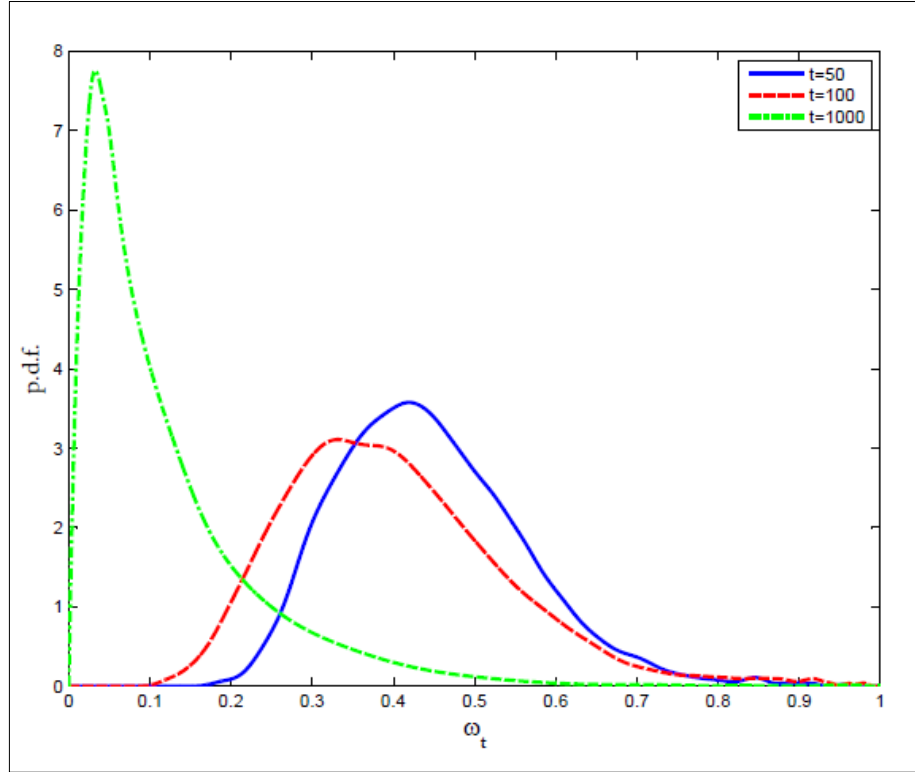


Figure 1.3 Survival and Price Impacts when $EIS_{EZ} = EIS_{LA} = 1.5$

Figure 1.3 graphs the probability density functions (p.d.f.s) of the LA-investor's wealth shares (ω_t) at times $t = 50, 100, 1000$ when $EIS_{EZ} = EIS_{LA} = 1.5$. The p.d.f.s are estimated non-parametrically from 5000 simulated data. At time 0, each investor has half of the aggregate wealth; that is, $\omega_0 = 0.5$. The preference parameters are $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.98$, $\lambda = 2.25$ and $b_{LA} = 0.02$. The technology parameters are fixed at the values from Table 1.1.

When $EIS_{EZ} = EIS_{LA} < 1$, the analysis in Subsection 1.4.1 suggests that the LA-investor saves more than the EZ-investor and thus favors his survival. As a result, two forces are at play here: portfolio decisions are against the LA-investor's wealth accumulation, while consumption decisions benefit it. It thus becomes nontrivial to explore whether the saving force is strong enough to reverse the result in the previous subsection. Table 1.4 presents the distributions of ω_t and $priceimpact_t$ for the case of $EIS_{EZ} = EIS_{LA} = 0.5$. The other parameter values are identical to those used in Table 1.3. Again, Panel A has a relative risk-aversion coefficient of 1, while Panel B has a relative risk-aversion coefficient of 3. Comparing Table 1.4 with Table 1.3, one can find that the results are almost identical, implying that in calibrated economies, the difference in saving behaviors induced by a small EIS is not large enough to help the LA-investor survive in the long run.

Table 1.4: Survival and Price Impacts: $EIS = 0.5$

Table 1.4 reports the distributions of the LA-investor's wealth shares (ω_t) and his price impacts ($priceimpact_t$) at times $t = 50$, 100 , and 1000 years when the LA-investor has initial wealth shares of $\omega_0 = 0.1$, 0.5 , and 0.9 and both investors have a relative risk aversion coefficient of 1 (Panel A) or 3 (panel B). Both investors have an EIS of 0.5: $EIS_{EZ} = EIS_{LA} = 0.5$. The other preference parameters are $\beta_{EZ} = \beta_{LA} = 0.98$, $\lambda = 2.25$ and $b_{LA} = 0.02$. The technology parameters are fixed at the values in Table 1.1. Each entry in Table 1.4 has three elements, corresponding, respectively, to the 5%, 50%, and 90% percentiles of the distributions of ω_t or $priceimpact_t$. The quantiles are estimated from the 5000 simulated sample paths at time t .

Panel A: $RA_{EZ} = RA_{LA} = 1$ ($\zeta_{EZ} = \zeta_{LA} = 0$)			
5%, 50% and 95% Quantiles of ω_t			
	$\omega_0 = 0.1$	$\omega_0 = 0.5$	$\omega_0 = 0.9$
$t = 50$	(.0645 .0942 .1479)	(.3153 .4572 .6899)	(.6380 .8726 .9708)
$t = 100$	(.0503 .0988 .1802)	(.2425 .4413 .7732)	(.4859 .8585 .9844)
$t = 1000$	(.0096 .0636 .3630)	(.0375 .2037 .9159)	(.0663 .3528 .9987)
5%, 50% and 95% Quantiles of $priceimpact_t$			
	$priceimpact_0 = .0314$	$priceimpact_0 = .2628$	$priceimpact_0 = .7364$
$t = 50$	(.0195 .0293 .0488)	(.1266 .2246 .5017)	(.4236 .6982 .8866)
$t = 100$	(.0150 .0309 .0615)	(.0889 .2115 .6143)	(.2497 .6815 .9305)
$t = 1000$	(.0028 .0192 .1554)	(.0111 .0714 .7628)	(.0201 .1489 .9913)
Panel B: $RA_{EZ} = RA_{LA} = 3$ ($\zeta_{EZ} = \zeta_{LA} = -2$)			
5%, 50% and 95% Quantiles of ω_t			
	$\omega_0 = 0.1$	$\omega_0 = 0.5$	$\omega_0 = 0.9$
$t = 50$	(.0576 .0902 .1314)	(.3466 .4692 .5715)	(.8387 .8899 .9207)
$t = 100$	(.0432 .0782 .1415)	(.2718 .4271 .5907)	(.7938 .8746 .9257)
$t = 1000$	(.0011 .0072 .0437)	(.0063 .0370 .2189)	(.0445 .2752 .7487)
5%, 50% and 95% Quantiles of $priceimpact_t$			
	$priceimpact_0 = .1096$	$priceimpact_0 = .5197$	$priceimpact_0 = .8985$
$t = 50$	(.0612 .0981 .1475)	(.3762 .4912 .5855)	(.8378 .8884 .9193)
$t = 100$	(.0453 .0843 .1601)	(.3030 .4522 .6031)	(.7941 .8731 .9243)
$t = 1000$	(.0011 .0074 .0458)	(.0064 .0387 .2492)	(.0467 .3064 .7508)

To better understand the effect of savings, Figure 1.4 plots the consumption policies for both investors in its left panel for the parameter configuration of Panel A of Table 1.4. Figure 1.4 also plots the equilibrium risk-free rate function in its right panel because, as suggested by the analysis in Subsection 1.4.1, the risk-free rate is the LA-investor's favored asset and thus largely determines his future utility. Two notable observations emerge. First, the difference in the endogenous saving ratios of the two investors is small. The maximum difference is 64 basis points, which is achieved when the LA-investor controls the total wealth. This small magnitude of the difference in saving ratios partly accounts for its difficulty in overcoming the disadvantage coming from the portfolio positions of the LA-investor in terms of wealth accumulation. The second observation is that as the LA-investor's wealth share decreases, so does the difference in the saving ratios of the two investors. This further weakens the effect of the saving difference on changing the LA-investor's survival prospect, as once his wealth-eroding investment positions start to reduce his wealth share, he also saves less, making his situation even worse in terms of wealth accumulation. The reason underlying the second observation is the following. As ω_t decreases, the EZ-investor controls more wealth, and because she saves less relative to the LA-investor, the risk-free rate increases, which in turn raises the LA-investor's future income because he tends to invest in the risk-free rate and, as a result of anticipating this rise in the permanent income, he consumes more.

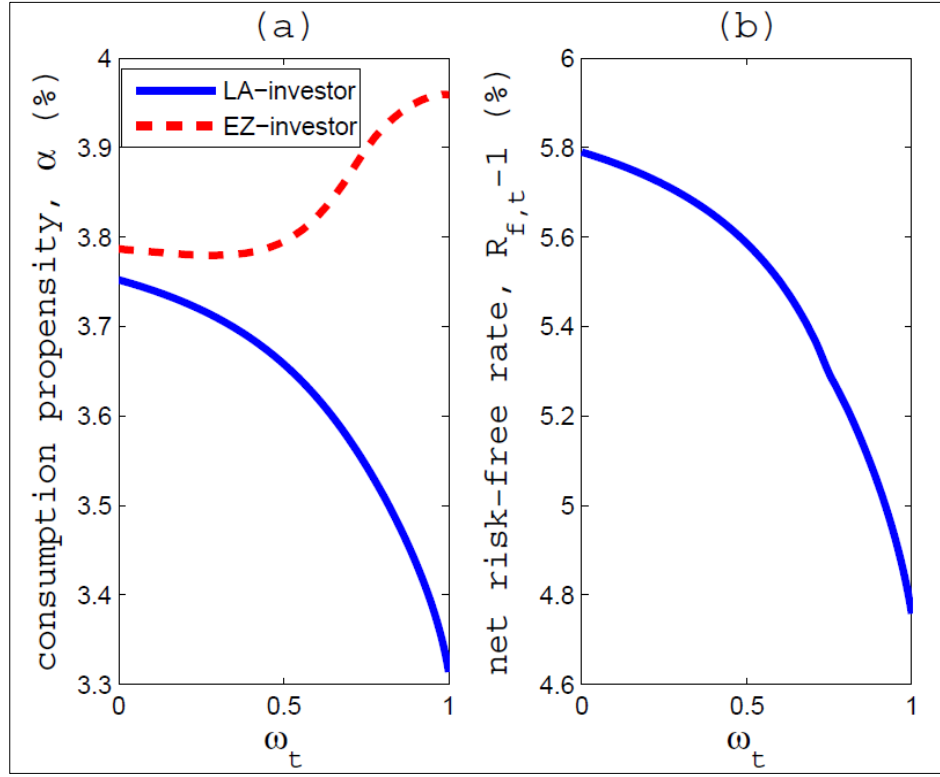


Figure 1.4 Consumption Policy and Risk-Free Rate when $EIS_{EZ} = EIS_{LA} = 0.5$

Figure 1.4 displays the consumption policies for both investors and the equilibrium risk-free rate function when both investors have a common small EIS, that is, when $EIS_{EZ} = EIS_{LA} = 0.5$. The preference parameters are $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.98$, $\lambda = 2.25$ and $b_{LA} = 0.02$. The technology parameters are fixed at the values in Table 1.1.

One might think that an increase in the relative importance of loss-aversion utility in the LA-investor's preference might change the LA-investor's survival prospects, as the difference in investors' saving ratios increases with b_{LA} , as suggested by the results in the representative-agent economies. To address this concern, Figure 1.5 depicts the dynamics of the distributions of ω_t in

Panel (a), as well as the consumption policies of both investors in Panel (b), for an economy that is otherwise identical to the one in Panel A of Table 1.3 except with a larger value of b_{LA} ($b_{LA} = 0.2$). To increase the chance of the LA-investor's survival, Panel (a) assumes that the LA-investor has 90% of the total wealth at time 0. Panel (b) indeed shows that the increase in b_{LA} enlarges the difference in the two investors' saving behavior, with a maximum of 3.5 percent achieved at $\omega_t = 1$. However, Panel (a) suggests that the LA-investor still vanishes in the long run, as the p.d.f.s shift to the left as time passes. This occurs for two reasons. First, the increase in b_{LA} also aggravates the disadvantage of the LA-investor in his portfolio positions, as he is now more risk averse and deviates more from the log utility than the EZ-investor. Second, although the maximum difference in saving ratios is impressive, this difference shrinks sharply, as the LA-investor loses wealth over time. Instead of raising b_{LA} , another possibility for the LA-investor to beat the EZ-investor is to decrease EIS or to increase λ , the parameter that determines the LA-investor's sensitivity to losses. Actually, this does not work for the same reason as in the case of raising b_{LA} . To save space, such an analysis is not reported here.

In sum, if the LA-investor differs from the EZ-investor only in a way such that he derives loss-aversion utility, then for empirically relevant parameter values, he will lose his wealth share in the long run, and his price impacts diminish along the way.

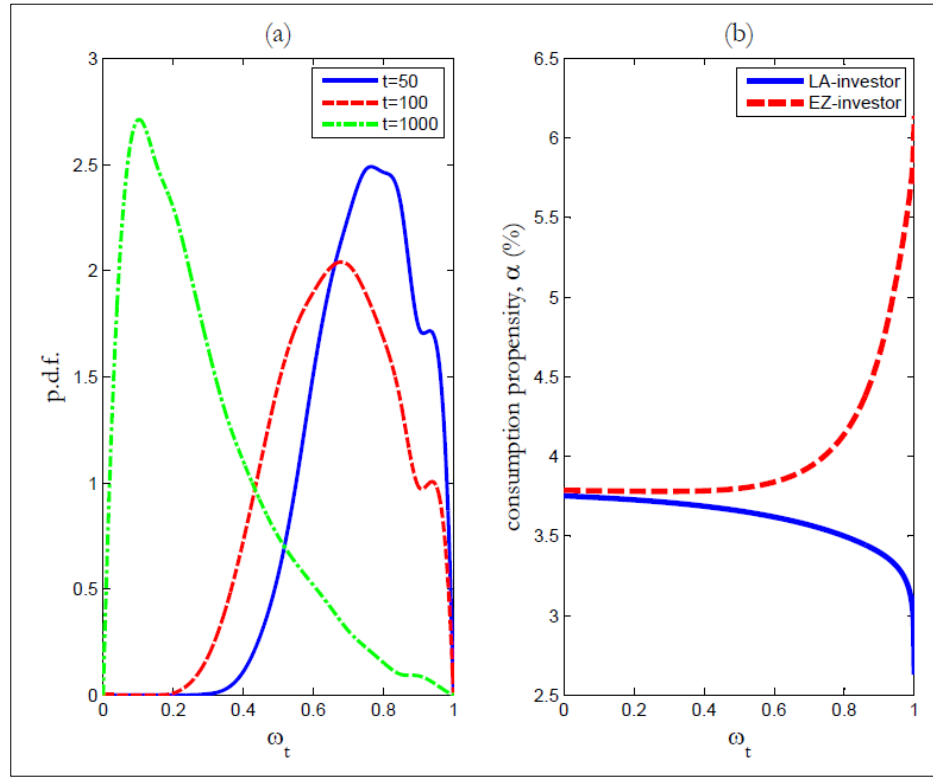


Figure 1.5 Survival and Consumption Policies when $b_{LA} = 0.2$ and $EIS_{EZ} = EIS_{LA} = 0.5$

Figure 1.5 depicts the probability density functions (p.d.f.s) of the LA-investor's wealth shares (ω_t) at times $t = 50, 100, 1000$, as well as the consumption policies of both investors when $b_{LA} = 0.2$ and $EIS_{EZ} = EIS_{LA} = 0.5$. The other preference parameters are $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.98$ and $\lambda = 2.25$. The technology parameters are fixed at the values in Table 1.1. The p.d.f.s are estimated non-parametrically from 5000 simulated data. At time 0, the LA-investor has a wealth share of 90%; that is, $\omega_0 = 0.9$.

1.5 Multi-Dimensional Heterogeneity in Preferences

So far, my analysis has assumed that the LA-investor and the EZ-investor are different only in one dimension: the LA-investor derives loss-aversion utility, while the EZ-investor does not. However, it is highly likely that they are also

different in other dimensions. This raises the question of how robust the result that the LA-investor vanishes in calibrated economies is to the introduction of additional differences in the investors' preferences.

Before examining the effect of the multi-dimensional heterogeneity of preferences on survival, I briefly discuss what kind of heterogeneity might be plausible in reality. In principle, on top of loss-aversion utility, investors can be different in the following three dimensions: risk aversion (parameter ζ), EIS (parameter ρ), and time preference (parameter β). As discussed in the previous section, risk aversion might not be a good candidate, as the very reason why the literature introduces loss aversion is to increase the LA-investor's risk aversion, which serves to generate a high equity premium. Therefore, to make the analysis empirically relevant, any perturbation of the risk-aversion parameter ζ should not reverse the order of the investors' risk attitude and would not change the survival prospects of the LA-investor. However, in the existing literature, researchers have not reached a consensus regarding the reasonable value for the EIS or the time discount rate. Some studies estimate the EIS to be well above 1 (e.g., Hansen and Singleton, 1982; Attanasio and Weber, 1989; Guvenen, 2001; Vissing-Jorgensen, 2002), while others estimate it to be well below 1 (e.g., Hall, 1988; Epstein and Zin, 1991; Campbell, 1999). Similarly, the calibrations of the time-patience parameter β are widely dispersed, ranging from 0.89 (Campbell and Cochrane, 1999) to 1.1 (Brennan and Xia, 2001). Therefore, in the following, I investigate the effect of a differing EIS or a differing time-patience factor on the LA-investor's survival.

1.5.1 Different EIS Parameters

To set the deck against the EZ-investor, I need to assume that the LA-investor has a larger EIS than the EZ-investor, so that he would save more than the EZ-investor in a growing economy. Specifically, I set $EIS_{EZ} = 0.5$, and vary EIS_{LA} to see when the LA-investor will survive for an economy with the technology parameters fixed at the values in Table 1.1 and the other preference parameters fixed at $\beta_{EZ} = \beta_{LA} = 0.98$, $RA_{EZ} = RA_{LA} = 1$, $\lambda = 2.25$ and $b_{LA} = 0.02$. It turns out that when $EIS_{LA} = 0.7$, the LA-investor starts to dominate the economy. The result is driven by the different saving behaviors induced by the different EIS. Figure 1.6 displays the dynamics of ω_t in Panel (a) and the consumption policies in Panel (b) assuming that $\omega_0 = 0.5$. Panel (a) shows that the p.d.f.s shift to the right as time passes, suggesting that the LA-investor tends to dominate the market in the long run. Panel (b) shows that the difference in the consumption ratios does not drop much when ω_t declines from intermediate levels of ω_t , because when ω_t decreases, the EZ-investor consumes more as a result of a strong income effect of the raised risk-free rate. Therefore, when the wealth share of the LA-investor declines due to his portfolio decisions, his advantage in terms of saving behavior will help him.

I have also tried other perturbations, and the result is qualitatively similar, although if I start from a larger value of EIS_{EZ} , a larger difference between EIS_{LA} and EIS_{EZ} is needed to make the LA-investor survive in the long run. For instance, when the EZ-investor has an EIS of 0.8, the critical level for EIS_{LA} increases to 1.5. This is because when EIS_{EZ} is large, the EZ-investor's saving rate has already been very high in a growing economy, leaving very little room for the LA-investor to improve. Whether the differences in investors'

EIS parameters are reasonable is an empirical issue and is subjective. The point of the present paper is to provide a framework that can be used to analyze under what conditions the LA-investor survives and his pricing impact persists.

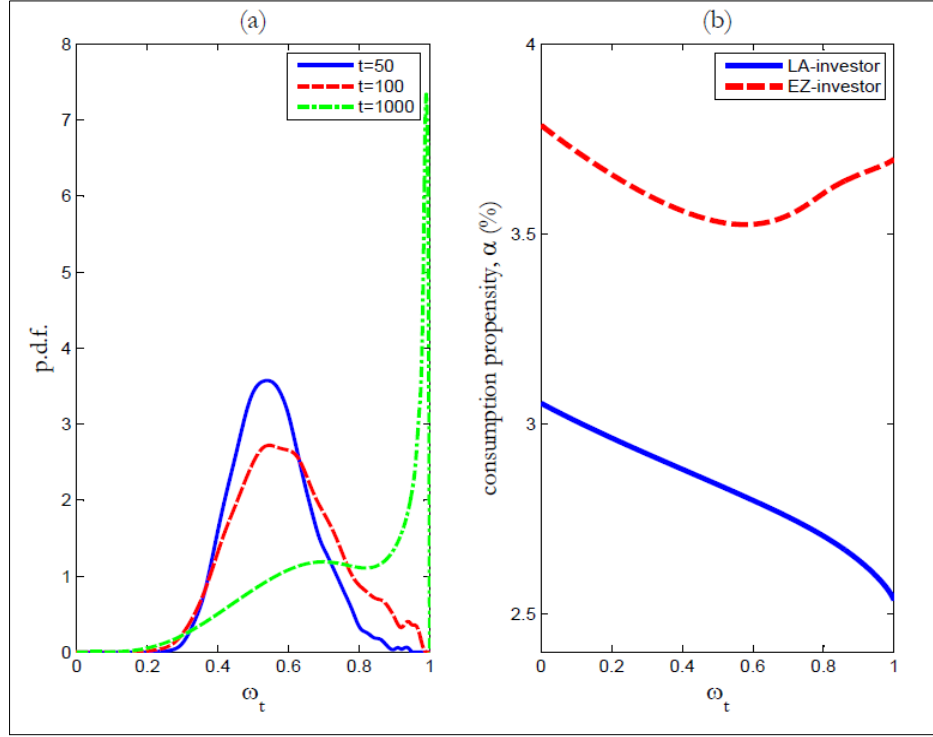


Figure 1.6 Survival and Consumption Policies when $EIS_{EZ} < EIS_{LA}$
($EIS_{EZ} = 0.5$ and $EIS_{LA} = 0.7$)

Figure 1.6 depicts the probability density functions (p.d.f.s) of the LA-investor's wealth shares (ω_t) at times $t = 50, 100, 1000$, as well as the consumption policies of both investors when $EIS_{EZ} = 0.5$ and $EIS_{LA} = 0.7$. The other preference parameters are $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.98$, $\lambda = 2.25$ and $b_{LA} = 0.02$. The technology parameters are fixed at the values in Table 1.1. The p.d.f.s are estimated non-parametrically from 5000 simulated data. At time 0, each investor has half of the aggregate wealth; that is, $\omega_0 = 0.5$.

1.5.2 Different Time Patience Parameter β

I conduct a similar exercise as in examining the effect of different EIS parameters. Specifically, I set $\beta_{LA} = 0.98$ and vary β_{EZ} to examine when the LA-investor dominates the market in the long run. The survival result is very sensitive to the time discount rate: a slight difference in β as small as two percent can overturn the effect of the LA-investor's portfolio decisions on his survival prospects. To illustrate this sensitivity, I set $EIS_{EZ} = EIS_{LA} = 1.5$, which means that the deck is set against the LA-investor, as he would consume more than the EZ-investor if they had a common β . Other preference parameters are fixed at $RA_{EZ} = RA_{LA} = 1$, $\lambda = 2.25$ and $b_{LA} = 0.02$, and the technology parameters are fixed at the values in Table 1.1. The result is robust to different relative risk aversions. Figure 1.7 depicts the dynamics of the distributions of ω_t and investors' consumption propensities when the LA-investor has half of the total wealth at time 0. Panel (a) shows that, as time passes, the p.d.f.s of ω_t shift to the right, suggesting that the LA-investor is accumulating wealth at a faster rate than the EZ-investor. Panel (b) displays the large difference in the endogenous saving ratios induced by the time-patience parameter. The minimum of this difference is 1.88%, and the maximum is 2.86%. These large magnitudes account for the LA-investor's eventual prosperity.

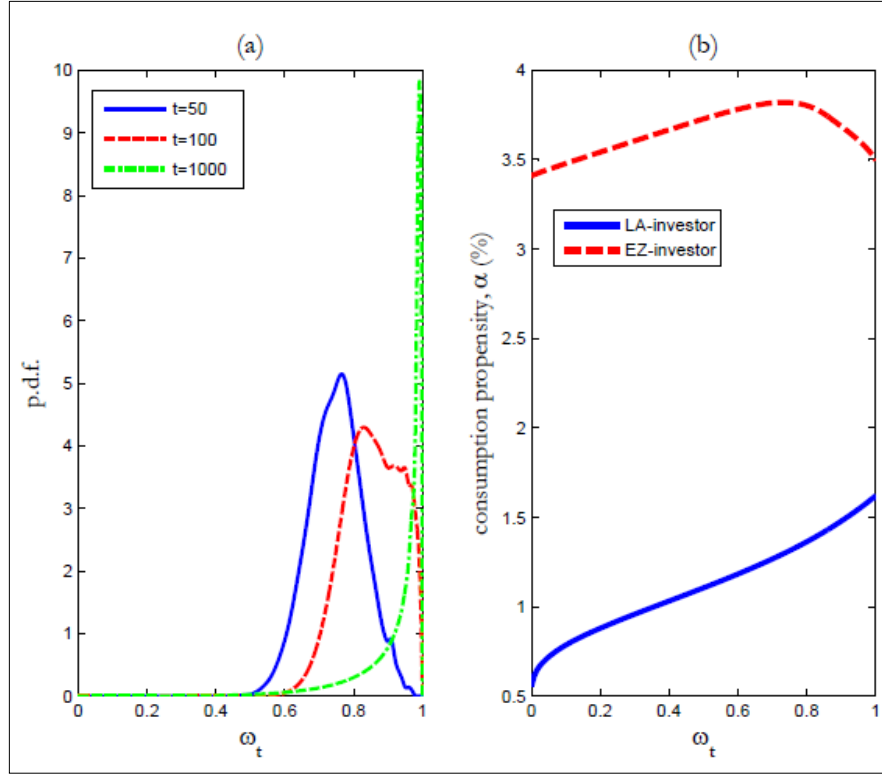


Figure 1.7 Survival and Consumption Policies when $\beta_{EZ} < \beta_{LA}$
($\beta_{EZ} = 0.96$ and $\beta_{LA} = 0.98$)

Figure 1.7 depicts the probability density functions (p.d.f.s) of the LA-investor's wealth shares (ω_t) at times $t = 50, 100, 1000$, as well as the consumption policies of both investors when $\beta_{EZ} = 0.96$ and $\beta_{LA} = 0.98$. The other preference parameters are $EIS_{EZ} = EIS_{LA} = 1.5$, $RA_{EZ} = RA_{LA} = 1$, $\lambda = 2.25$ and $b_{LA} = 0.02$. The technology parameters are fixed at the values in Table 1.1. The p.d.f.s are estimated non-parametrically from 5000 simulated data. At time 0, each investor has half of the aggregate wealth; that is, $\omega_0 = 0.5$.

1.6 Conclusion

This paper studies the survival and price impact of loss-averse investors in a financial economy in which arbitrageurs have Epstein-Zin preferences. I obtain two main results. First, if the LA-investor differs from the EZ-investor only in the way of deriving loss-aversion utility, then the LA-investor will be driven out of the market and thus will have no effect on long-run asset prices for an empirically relevant range of parameters. This result is driven by the distorted portfolio decisions induced by loss aversion, which makes the LA-investor act in a way that is more different from the log investor than the EZ-investor. Second, once additional heterogeneity is recognized in investors' preferences, for example, when they are heterogeneous with respect to their EIS parameter or time-patience factors, then the first result can be easily overturned by the different equilibrium saving behaviors induced by this new heterogeneity. Therefore, my paper provides a framework with which to quantify the effect of heterogeneous preferences on asset prices and the long-run wealth of investors in a dynamic financial market.

This research does not end the discussion, however. Several open questions remain. First, this paper has assumed a complete market structure. The survival result might be very different in an incomplete market, as suggested by the existing literature (e.g., Blume and Easley, 2006; Cao, 2009). It would thus be interesting to compute how market incompleteness would change the results. Second, this paper focuses only on the loss-aversion feature of prospect theory and ignores its two other features, namely, diminishing sensitivity and probability weighting. The literature has shown that both features help to explain certain financial phenomena. For example, Li and

Yang (2008) show that diminishing sensitivity can generate price momentum, while Barberis and Huang (2008) argue that probability weighting leads to the overpricing of positively skewed securities. It would also be interesting to examine the survival and price impacts of an investor whose preference has all three features of prospect theory. Third, the loss-averse investors are homogeneous in my model. It is likely that even loss-averse investors are heterogeneous in a number of ways: in the degree of sensitivity to losses (parameter λ), in the relative importance of loss aversion utility in their preferences (parameter b)¹² or in the reference levels that determine their gains/losses. I leave all these interesting questions for future research.

¹²In fact, the difference between the LA-investor and the EZ-investor in my model can be viewed as an extreme case of differing b : $b_{EZ} = 0$ and $b_{LA} > 0$.

APPENDIX

This appendix sketches the procedure used to numerically solve the model in Chapter 1. I focus on the non-unit EIS case ($\rho_i \neq 0$), and the solution procedure for the unit EIS case is slightly different. The algorithm is developed based on Kubler and Schmedders (2003) and is summarized as follows:

Step 0: Define a finite grid on $[0,1]$. Choose two continuous functions, $\alpha_{EZ}^0(\cdot)$ and $\alpha_{LA}^0(\cdot)$, as initials for the investors' consumption policy functions. These initials define the initial for the price-dividend ratio function, $f^0(\cdot)$, through equation (22). Then on each grid point ω_t , go through steps 1-4.

Step 1: Given functions $\alpha_{EZ}^n(\cdot)$ and $\alpha_{LA}^n(\cdot)$, suppose that the LA-investor allocates nothing on the stock; that is, $s_{LA}^{(n+1)}(\omega_t) = 0$. Then use both investors' value functions, equation (13), the EZ-investor's first-order equation, (17), the state transition functions, and equation (27), to solve five unknowns: $\alpha_{EZ,t}^*$, $\alpha_{LA,t}^*$, $R_{f,t}$, $\omega_{t+1,H}$, $\omega_{t+1,L}$, where $\omega_{t+1,H}$ and $\omega_{t+1,L}$ are the next-period wealth shares when $\theta_{t+1} = \theta_H$ and θ_L , respectively.

Step 2: Plug the solved $\alpha_{LA,t}^*$, $R_{f,t}$, $\omega_{t+1,H}$ and $\omega_{t+1,L}$ into equations (14) and (15) to get $FOC_{LA,+}$ and $FOC_{LA,-}$. If $FOC_{LA,+} \leq 0$ and $FOC_{LA,-} \geq 0$, then set $\alpha_{EZ}^{n+1}(\omega_t) = \alpha_{EZ,t}^*$ and $\alpha_{LA}^{n+1}(\omega_t) = \alpha_{LA,t}^*$. If $FOC_{LA,+} > 0$, then go to Step 3; otherwise, go to Step 4.

Step 3: Use both investors' value functions, equation (13), the EZ-investor's first-order equation, (17), the LA-investor's first-order condition for a positive

investment, equation (14), the state transition functions, and equation (27), to solve six unknowns: $\alpha_{EZ,t}^*$, $\alpha_{LA,t}^*$, $R_{f,t}$, $\omega_{t+1,H}$, $\omega_{t+1,L}$, $s_{LA,t}^*$. Set $\alpha_{EZ}^{n+1}(\omega_t) = \alpha_{EZ,t}^*$ and $\alpha_{LA}^{n+1}(\omega_t) = \alpha_{LA,t}^*$.

Step 4: Use both investors' value functions, equation (13), the EZ-investor's first-order equation, (17), the LA-investor's first-order condition for a negative investment, equation (15), the state transition functions, equation (27), to solve six unknowns: $\alpha_{EZ,t}^*$, $\alpha_{LA,t}^*$, $R_{f,t}$, $\omega_{t+1,H}$, $\omega_{t+1,L}$, $s_{LA,t}^*$. Set $\alpha_{EZ}^{n+1}(\omega_t) = \alpha_{EZ,t}^*$ and $\alpha_{LA}^{n+1}(\omega_t) = \alpha_{LA,t}^*$.

Step 5: Check whether the following stop criterion is satisfied:

$$\max_{\omega_t} \left\| \left(\alpha_{EZ}^{n+1}(\cdot), \alpha_{LA}^{n+1}(\cdot), f^{n+1}(\cdot) \right) - \left(\alpha_{EZ}^n(\cdot), \alpha_{LA}^n(\cdot), f^n(\cdot) \right) \right\| < \tau,$$

where τ is an error tolerance. If yes, then the algorithm terminates, and the next step is to set the consumption and investment policy functions and risk-free rate function as those solved in the last round. Otherwise, increase n by 1 and go to Step 1.

In the implementation of the algorithm, I divide $[0,1]$ into 150 grid points and set the tolerance level at 10^{-7} . Kubler and Schmedders (2003) provide a method to assess the accuracy of a candidate solution by computing the maximal relative error in Euler equations. In my computations, the maximum errors lie below 10^{-6} , suggesting that the algorithm produces reasonably accurate solutions.

REFERENCES

- Alchian, Armen, 1950, Uncertainty Evolution and Economic Theory, *Journal of Political Economy* 58, 211--221.
- Attanasio, P. Orazio, and Guglielmo Weber, 1989, Intertemporal Substitution, Risk Aversion and the Euler Equation for Consumption, *Economic Journal* 99, 59--73.
- Bansal, Ravi, and Amir Yaron, 2004, Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles, *Journal of Finance* 59, 1481-1509.
- Barberis, Nicholas, and Ming Huang, 2001, Mental Accounting, Loss Aversion, and Individual Stock Returns, *Journal of Finance* 56, 1247-1292.
- Barberis, Nicholas, and Ming Huang, 2007, The Loss Aversion/Narrow Framing Approach to Stock Market Pricing and Participation Puzzles, in: Rajnish Mehra (ed.), *Handbook of the Equity Risk Premium*, Elsevier, Amsterdam. chapter 18, 199-229.
- Barberis, Nicholas, and Ming Huang, 2008, Stocks as Lotteries: The Implications of Probability Weighting for Security Prices, *American Economic Review*, 98:5, 2066--2100.
- Barberis, Nicholas, and Ming Huang, 2009, Preferences with Frames: A New Utility Specification that Allows for the Framing of Risks, *Journal of Economic Dynamics and Control* 33, 1555-1576.
- Barberis, Nicholas, Ming Huang, and Tano Santos, 2001, Prospect Theory and Asset Prices, *Quarterly Journal of Economics* 116, 1-53.
- Barberis, Nicholas, Ming Huang, and Richard Thaler, 2006, Individual Preferences, Monetary Gambles, and Stock Market Participation: A Case

- for Narrow Framing, *American Economic Review* 96, 1069-1090.
- Barberis, Nicholas and Richard Thaler, 2003, A Survey of Behavioral Finance, in: George M. Constantinides, Milton Harris and René Stulz (ed.), *Handbook of the Economics of Finance*, edition 1, volume 1, chapter 18, 1053-1128 Elsevier.
- Barberis, Nicholas and Wei Xiong, 2009, What Drives the Disposition Effect? An Analysis of a Long-standing Preference-based Explanation, *Journal of Finance* 64, 751-784.
- Benartzi, Shlomo and Richard Thaler, 1995, Myopic Loss Aversion and the Equity Premium Puzzle, *Quarterly Journal of Economics* 110, 73-92.
- Berkelaar, Arjan, and Roy Kouwenberg, 2009, From Boom `Til Bust: How Loss Aversion Affects Asset Prices, *Journal of Banking & Finance*, Volume 33, Issue 6, 1005-1013.
- Berkelaar, Arjan, Roy Kouwenberg, and Thierry Post, 2004, Optimal Portfolio Choice under Loss Aversion, *Review of Economics and Statistics* 86, 973-987.
- Blume, Lawrence, and David Easley, 1992, Evolution and Market Behavior, *Journal of Economic Theory* 58, 9--40.
- Blume, Lawrence, and David Easley, 2006, If You're So Smart, Why Aren't You Rich? Belief Selection in Complete and Incomplete Markets, *Econometrica* 74, 929--966.
- Borovička, Jaroslav, 2009, Heterogeneous Beliefs under Recursive Preferences, University of Chicago, working paper.
- Brennan, Michael, and Yihong Xia, 2001, Stock Price Volatility and Equity Premium, *Journal of Monetary Economics* 47, 249--283.
- Campbell, John Y., 1999, Asset prices, consumption and the business cycle,

- in John B. Taylor, and Michael Woodford, eds.: *Handbook of Macroeconomics*, vol. 1 (Elsevier Science, North-Holland, Amsterdam).
- Campbell, John, and John Cochrane, 1999, By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior, *Journal of Political Economy* 107, 205--251.
- Cao, Dan, 2009, Collateral Shortages, Asset Price and Investment Volatility with Heterogeneous Beliefs, MIT working paper.
- Cogley, Timothy, and Thomas J. Sargent, 2009. Diverse Beliefs, Survival and the Market Price of Risk, *Economic Journal*, Royal Economic Society, vol. 119 (536), 354-376.
- Condie, Scott, 2008, Living with Ambiguity: Prices and Survival when Investors Have Heterogeneous Preferences for Ambiguity, *Economic Theory* 36:81--108.
- Coury, Tarek and Emanuela Sciubba, 2005, Disagreeing Forever: Survival and Belief Heterogeneity in Incomplete Markets, working paper, University of Oxford and University of Cambridge.
- De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldman, 1991, The Survival of Noise Traders in Financial Markets, *Journal of Business* 64, 1--19.
- Espstein, Lawrence, and Stanley Zin, 1989, Substitution, Risk Aversion, and the Temporal Behavior of Asset Returns: A Theoretical Framework, *Econometrica* 57, 937-969.
- Espstein, Lawrence, and Stanley Zin, 1991, Substitution, Risk Aversion, and the Temporal Behavior of Asset Returns: An Empirical Analysis, *The Journal of Political Economy*, Volume 99, Issue 2, 263-286.
- Friedman, Milton, 1953, The Case for Flexible Exchange Rates, in *Essays in*

- Positive Economics*, University of Chicago Press, Chicago.
- Gallmeyer, Michael, and Burton Hollifield, 2008, An Examination of Heterogeneous Beliefs with a Short-Sale Constraint in a Dynamic Economy, *Review of Finance* 12(2):323-364.
- Gomes, Francisco, 2005, Portfolio Choice and Trading Volume with Loss Averse Investors, *Journal of Business* 78, 675-706.
- Grünea, Lars, and Willi Semmler, 2008, Asset Pricing with Loss Aversion, *Journal of Economic Dynamics and Control*, Volume 32, Issue 10, 3253-3274.
- Guvenen, Fatih, 2001, Mismeasurement of the Elasticity of Intertemporal Substitution: The Role of Limited Stock Market Participation, Unpublished manuscript, University of Rochester.
- Hakansson, Nils H., 1971, Capital Growth and the Mean-Variance Approach to Portfolio Selection, *Journal of Financial and Quantitative Analysis* 6, 517-555.
- Hall, Robert E., 1988, Intertemporal Substitution in Consumption, *Journal of Political Economy* 96, 339--357.
- Hansen, Lars P., and Ravi Jagannathan, 1991, Restrictions on Intertemporal Marginal Rates of Substitutions Implied by Asset Returns, *Journal of Political Economy*, 225--262.
- Hansen, Lars P., and Kenneth Singleton, 1982, Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models, *Econometrica* 50, 1269--1286.
- Hansen, Lars P., and Kenneth Singleton, 1983, Stochastic Consumption, Risk Aversion and the Temporal Behavior of Asset Returns, *Journal of Political Economy*, 249--265.

- Heaton, John, and Deborah Lucas, 1996, Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing, *Journal of Political Economy* 104, 443--487.
- Kahneman, Daniel, 2003, Maps of Bounded Rationality: Psychology for Behavioral Economics, *American Economic Review* 93, 1449--1475.
- Kahneman, Daniel, Lovallo, D., 1993, Timid Choices and Bold Forecasts: A Cognitive Perspective on Risk Taking, *Management Science* 39, 17--31.
- Kahneman, Daniel, and Amos Tversky, 1979, Prospect Theory: An Analysis of Decision under Risk, *Econometrica* 47, 263-291.
- Kocherlakota, N.R., 1996, The Equity Premium: It's Still a Puzzle, *Journal of Economic Literature* 34, 42-71.
- Kubler, Felix, and Karl Schmedders, 2003, Stationary Equilibria in Asset-Pricing Models with Incomplete Markets and Collateral, *Econometrica* 71, 1767--95.
- Kyle, Albert, Hui Ou-Yang, and Wei Xiong, 2006, Prospect Theory and Liquidation Decisions, *Journal of Economic Theory* 129, 273-285.
- Kyle, Albert, and Albert Wang, 1997, Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test?, *Journal of Finance* 52, 2073--2090.
- Leonid Kogan, Stephen Ross, Jiang Wang, and Mark M. Westerfield, 2009, Market Selection, MIT working paper.
- Li, Yan and Liyan Yang, 2008, Prospect Theory, the Disposition Effect, and Asset Prices, University of Toronto, working paper.
- Mailath, George, and Alvaro Sandroni, 2003, Market Selection and Asymmetric Information, *Review of Economic Studies* 70, 343--369.
- McQueen, Grant, and Keith Vorkink, 2004, Whence GARCH? A Preference-

- Based Explanation for Conditional Volatility, *Review of Financial Studies*, 17(4):915-949.
- Mehra, Rajnish, and Edward Prescott, 1985, The Equity Premium Puzzle, *Journal of Monetary Economics*, 145--161.
- Palomino, Frederic, 1996, Noise Trading in Small Markets, *Journal of Finance* 51, 1537--1550.
- Sandroni, Alvaro, 2000, Do Markets Favor Agents Able to Make Accurate Predictions?, *Econometrica* 68, 1303--134.
- Samuelson, Paul, 1977, St. Petersburg Paradoxes, *Journal of Economic Literature* 15, 24-55.
- Sandroni, Alvaro, 2005, Market Selection When Markets Are Incomplete, *Journal of Mathematical Economics* 41, 91--104.
- Sciubba, Emanuela, 2005, Asymmetric Information and Survival in Financial Markets, *Economic Theory* 25 353--379.
- Vissing-Jorgensen, Annette, 2002, Limited Asset Market Participation and the Elasticity of Intertemporal Substitution, *Journal of Political Economy* 110, 825--853.
- Weil, Philippe, 1989, The Equity Premium Puzzle and the Risk-Free Rate Puzzle, *Journal of Monetary Economics*, Volume 24, Issue 3, 401-421.
- Yan, Hongjun, 2008, Natural Selection in Financial Markets: Does It Work?, *Management Science* 54 (11), 1935-1950.

CHAPTER 2

PROSPECT THEORY, THE DISPOSITION EFFECT AND ASSET PRICES¹³

2.1 Introduction

One of the mostly studied individual trading behaviors is the disposition effect: investors have a greater tendency to sell assets that have risen in value since purchase than those that have fallen.¹⁴ This effect has been observed in many markets, both for retail investors and for professional investors.¹⁵ It is puzzling because none of the most obvious rational explanations, such as portfolio rebalancing or information story, can entirely account for the disposition effect (Odean, 1998). As a result, an alternative view based on prospect theory has gained favor.

¹³This chapter is based on a joint paper with Yan Li.

¹⁴Shefrin and Statman (1985) coined the term the disposition effect. This effect is puzzling because the purchase price of a stock should not matter much for an investor's decision to sell it. In addition, tax laws encourage investors to sell losers rather than winners to reduce taxes. In a careful further study, Odean (1998) finds that the most obvious explanations, namely those based on information, taxes, rebalancing, or transaction costs, fail to capture important features of the data.

¹⁵Odean (1998), Grinblatt and Keloharju (2001), and Feng and Seasholes (2005) find disposition effects in the stock markets of the U.S., Finland, and China, respectively. Grinblatt and Keloharju (2001), Shapira and Venetia (2001), Werners (2003), Garvey and Murphy (2004), Coval and Shumway (2005), Locke and Mann (2005), Frazzini (2006), and Scherbina and Jin (2006) document the disposition effect in the trading of professionals who trade on behalf of their firms; Genesove and Mayor (2001) and Heath *et al.* (1999) document disposition effects in the housing market and in the exercise of executive stock options, respectively. Webber and Camerer (1998) and Oehler *et al.* (2002) uncover disposition effects with experimental data. See Feng and Seasholes' (2005) Appendix A for more empirical studies on the disposition effect.

The literature has produced both *informal* arguments (e.g., Odean, 1998) and *formal* models (Kyle *et al.*, 2006; Hens and Vlcek, 2006; Barberis and Xiong, 2009), relying on prospect theory to explain the disposition effect. As a prominent theory of decision-making under risk, prospect theory was first proposed by Kahneman and Tversky (1979) and extended by Tversky and Kahneman (1992). A prospect theory investor evaluates gambles through gains and losses, not final wealth levels. The value function used by the investor to process gains and losses has a kink in the origin, indicating that investors are more sensitive to losses than to gains; this feature is referred to as *loss aversion* in the literature. Moreover, the value function is concave for gains and convex for losses, meaning that the investor is risk averse for gains and risk-loving for losses, which is known as *diminishing sensitivity*.¹⁶

Aside from using prospect theory to study the underlying cause of the disposition effect, recent empirical studies suggest that the disposition effect has pricing and volume implications: it can generate momentum in stock returns (Grinblatt and Han, 2005; Shumway and Wu, 2007), induce post-earnings announcement drift (Frazzini, 2006), and contribute to a positive correlation between returns and volumes (e.g., Statman *et al.*, 2006).

While existing studies have offered many insightful understandings on the link

¹⁶For a review of prospect theory, see Barberis and Thaler's (2003) Section 3.2.1 or Barberis and Huang's (2008) Section 2. Another salient feature of prospect theory is probability weighting: the investor overweights small probabilities and underweights intermediate probabilities in computing the expectation. We don't incorporate this feature in our model due to the reasons discussed in Section 2.2.

from prospect theory to the disposition effect, and on the link from the disposition effect to return and volume patterns, they have almost always investigated these two links separately. On the one hand, the partial equilibrium models proposed by Kyle *et al.* (2006), Hens and Vlcek (2006) or Barberis and Xiong (2009) assume an exogenous stock return process, and are therefore silent about the pricing and volume implications of the disposition effect. On the other hand, Grinblatt and Han's (2005) theoretical model shows that the disposition effect can lead to price momentum, but it begins with a demand function featuring the disposition effect without exploring whether such a demand function can indeed be generated from prospect theory preferences. In particular, Barberis and Xiong (2009)'s partial equilibrium results suggest that when the expected stock return is high, the disposition effect leads to a reversed disposition effect, implying a *reversal* in stock returns and a *negative* correlation between returns and volumes. The literature thus lacks a theoretical foundation to support the intuition from prospect theory to the disposition effect and the intuition from the disposition effect to price momentum or volume patterns.

Without such a general equilibrium model, the following questions are thus left unanswered: Whether the intuitions emphasized in existing studies are coherent in a unified framework? Does prospect theory predict the disposition effect when stock returns are endogenous? Which component of prospect theory drives the momentum, and which drives the reversal? In a calibrated economy, how much can prospect theory explain the data? The challenges of proposing such a general equilibrium model come from: (i) an investor's decision involves solving an optimal stopping time problem with a non-smooth

and partially convex objective function, and (ii) the state vector in the general equilibrium model is high-dimensional, including the distribution of stock holdings and purchase prices (i.e., the reference points) for all investors in every possible state of nature.

In this paper, we develop an overlapping-generation (OLG) model to simplify an investor's optimal stopping time problem and to reduce the dimensions of the state vector, making it possible to simultaneously study the link between prospect theory and the disposition effect, as well as the impact of this effect on stock prices. In our model, over their lifetimes, investors can trade stocks and a risk-free asset in the financial market, and, at the end of their final periods, receive prospect theory utility based on their trading profits. The behavior of those investors who bought stocks in previous periods can potentially exhibit the disposition effect. Our model shows that different components of prospect theory make different predictions regarding trading behavior, return predictability and volume patterns.

Specifically, the *diminishing sensitivity* component, which posits that investors are risk averse (risk-loving) for gains (losses), or that the value function is concave (convex) in the gain (loss) domain, predicts the disposition effect in equilibrium, which in turn drives price momentum and a positive correlation between returns and volumes (See Subsection 2.4.2).¹⁷ However, the *loss aversion* component, which says that investors are more sensitive to losses

¹⁷Throughout this paper, we follow the literature in using the terms diminishing sensitivity and concavity/convexity interchangeably to refer to the S-shaped value function of prospect theory.

than to gains, or that the value function has a kink at the origin, predicts exactly the opposite, namely, a reversed disposition effect in individual trading, reversal in the cross-section of stock returns and a negative correlation between returns and volumes (See Subsection 2.4.3). In a calibrated economy, when preference parameters are set at the values estimated by the previous studies, the concavity/convexity feature of prospect theory value function dominates, so that our model can generate an annual momentum of up to 7% (See Subsection 2.4.4).

The intuition for the implications of diminishing sensitivity is as follows. When a stock experiences good news and increases in value relative to the purchase price, these investors will be keen to sell it to lock in the paper gain, due to the concavity of the value function of prospect theory in the region of gains. Their selling increases volume. The selling pressure, moreover, depresses the stock price, generating subsequent higher returns. Similarly, when a stock experiences bad news and decreases in value relative to the purchase price, these investors are facing capital losses, and they are reluctant to sell, absent a premium, because of the convexity in the region of losses. In this case, the volume dries up, and the price is inflated, giving rise to subsequent lower returns. In this way, our model proves the internal consistency of the existing informal arguments which link prospect theory to the disposition effect (e.g., Odean, 1998) and which rely on the disposition effect to explain the momentum effect (e.g., Grinblatt and Han, 2005) and the positive relationship between price changes and volume (e.g., Odean, 1998; Statman *et al.*, 2006).

What's the intuition for the implications of loss aversion? Loss aversion means

that prospect theory value function has a kink at the origin, and investors are afraid of holding stocks if they are close to the kink. It is well understood in the literature that loss aversion can raise equity premiums in equilibrium (e.g., Benartzi and Thaler, 1995; Barberis *et al.*, 2001). So in equilibrium, good (bad) news will push investors far from (close to) the kink, making them more likely to hold (sell) stocks when facing gains (losses). This resulting reversed disposition effect, in turn, leads to a negative correlation between returns and volumes, as well as reversal in the cross-section of returns: when a stock experiences good (bad) news and increases (decreases) in value relative to the purchase price, investors, according to the reversed disposition effect, want to hold (sell) stocks, which reduces (raises) the trading volume and inflates (depresses) the stock price; from that higher (lower) base, subsequent stock returns will also be lower (higher).

To the best of our knowledge, this paper is the first to comprehensively study the implications of prospect theory for individual trading behavior, asset prices and trading volume in a dynamic setting. Previous research on the effect of prospect theory in the asset pricing literature has focused primarily on the loss aversion component and shown that it can increase the equity premium, i.e., the *mean* of stock returns in excess of the risk free rate (e.g., Barberis *et al.*, 2001).¹⁸ Our model demonstrates that loss aversion also has implications for return *predictability* and the correlation between returns and volumes. In addition, our paper shows that the S-shaped value function of prospect theory

¹⁸Recently, in a one period (two dates) model, Barberis and Huang (2008) show that the *probability weighting* feature of prospect theory can cause a security's individual skewness to be priced in equilibrium.

helps explain the disposition effect, the momentum effect and the comovement between stock returns and turnovers. Over and above these results, in Subsection 2.4.2, we argue that diminishing sensitivity alone, in the absence of loss aversion, can raise equity premiums.

The rest of the paper is organized as follows. Section 2.2 describes the model, and Section 2.3 characterizes the equilibrium. Section 2.4 solves the price-dividend ratios and uses simulated data to analyze the implications of diminishing sensitivity and loss aversion for individual trading behavior, asset prices and trading volumes. In particular, Subsection 2.4.4 conducts a quantitative analysis to evaluate how well our model matches the historical data. Section 2.5 concludes the paper. The appendix discusses the robustness of our results to certain modeling assumptions.

2.2. The Model

Let us consider an OLG model with one consumption good. Time is discrete and indexed by t . In each period, there are 3 generations (age-1, age-2 and age-3), each with a unitary mass. We adopt an OLG setup simply to reduce the dimension of the state vector. In the context of the disposition effect, the reference points usually relate to the purchase prices, which enter the state of the economy via the disposition effect, making the state history dependent. In an OLG setup, investors live for a finite period of time, so their purchase prices involve only a finite number of periods, effectively reducing the dimension of the state vector. The OLG setup should therefore not be interpreted literally. Generations should be understood as generations of transactions, not generations of people. Since the average holding periods of stocks are six

months to one year, one generation corresponds to six months to one year. Why are there three generations in each period? First, in order to study the disposition effect, which concerns selling decisions, we need at least three generations. In the standard two generation models, old investors always sell stocks whether facing good news or bad, thereby automatically ruling out the disposition effect. On the other hand, one model with more than two generations allows some investors to decide when to liquidate stocks which they bought in previous periods. Second, if there were more than three generations, the state vector would be highly dimensional, making the model intractable. In Appendix 2.A.2, we intuitively argue that our results might still hold in a setup with more than three generations.

2.2.1 Financial Assets

There are two traded assets: a risk-free bond and a risky stock. The bond is in perfectly elastic supply at a constant gross interest rate $R_f > 1$. The stock pays a random dividend $D_t > 0$ in period t . The dividend growth rate $\theta_{t+1} = \frac{D_{t+1}}{D_t}$ is *i.i.d.* over time, and follows a distribution given by

$$\theta_{t+1} = \begin{cases} \theta_H & \text{with probability } \frac{1}{2} \\ \theta_L & \text{with probability } \frac{1}{2} \end{cases}, \text{ with } 0 < \theta_L < \theta_H. \quad (1)$$

The stock is in limited supply (normalized as 1) and is traded in a competitive market at price P_t . Let R_{t+1} be the gross return on the stock between time t and $t+1$, i.e., $R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$.

Investors can buy or short bonds at any level, but they can not short stocks, and if they buy stocks, they can hold exactly 1 unit in each period. We assume

that people hold either zero or one unit of stock for several reasons. First, this specification is realistic in the sense that the lower (upper) bound of the holding position captures the shorting (borrowing) constraints in stock trading. Second, the assumption that people buy at most one unit of stock at one time captures the idea that they tend to form different mental accounts for the same stock bought at different prices. Third, a binary choice in stock holdings simplifies an investor's decisions, because otherwise it is very difficult to characterize the investor's demand function due to the convexity of the Kahneman and Tversky (1992) value function in the loss domain. Finally, a binary choice and an OLG setup combine to reduce the complicated optimal stopping problem of an age-2 investor owning a stock to a simple problem of choosing between an early liquidation and a late liquidation.

2.2.2 Beliefs

In order to study the impact of the disposition effect on trading volumes, we make two assumptions on investors' beliefs. First, investors hold heterogeneous beliefs about the dividend growth rate within one period. Due to this cross-sectional heterogeneity in beliefs, investors, in particular young investors, will make different investment decisions: more optimistic investors will purchase a stock, while more pessimistic investors will not. Second, an investor's one-period-ahead dividend forecast changes during his lifetime. The time-variation in an investor's belief will motivate the selling of a middle-aged investor who purchased the stock when he was young. With these two assumptions, we ensure that in each period, there is always a group of middle-aged investors who bought stocks last period and want to sell them this period. It is this group of investors that can potentially exhibit a disposition effect. Of

course, these two assumptions are just a modelling device, and any other trading motives, such as liquidity shocks (e.g., Kaustia, 2008), can also serve the same purpose.

As a matter of fact, in the informal arguments that have been used to link prospect theory and the disposition effect, investors are often assumed to experience belief changes, i.e., time-variation in an investor's beliefs is often maintained as the following quotation from Odean (1998, p. 1777) illustrates.

(S)uppose an investor purchases a stock that *she believes to have an expected return high enough* to justify its risk. If the stock appreciates and the investor continues to use the purchase price as a reference point, the stock price will then be in a more concave, more risk-averse, part of the investor's value function. It may be that the stock's expected return continues to justify its risk. However, if the investor *somewhat lowers her expectation of the stock's return*, she will be likely to sell the stock. What if, instead of appreciating, the stock declines? Then its price is in the convex, risk-seeking, part of the value function. Here the investor will continue to hold the stock even if its expected return falls lower than would have been necessary for her to justify its original purchase. Thus *the investor's belief about expected return must fall further* to motivate the sale of a stock that has already declined than one that has appreciated. [Emphasis added as italics]

Formally, in period t , investor i believes that the dividend growth rate θ_{t+1} follows a distribution given by

$$\theta_{t+1} = \begin{cases} \theta_H & \text{with probability } q_{i,t} \\ \theta_L & \text{with probability } 1 - q_{i,t} \end{cases}, \quad (2)$$

where $q_{i,t}$ is a random variable with uniform distribution on $[0,1]$ and $q_{i,t}$ is *i.i.d.* across investors (index i) and over time (index t).¹⁹ On average, investors have the correct beliefs, since the mean of $q_{i,t}$ is equal to $\frac{1}{2}$.

Investors are forward looking, so that we can apply the standard dynamic programming techniques to solve their optimal decision problems.

2.2.3 Preference

An investor derives prospect theory utility from trading assets in the spirit of Kahneman and Tversky (1979, 1992).²⁰ When investor i is born, he is endowed with $W_{1,i}$ units of consumption good. He can trade when he is young and middle-aged, leaving his final wealth as $W_{3,i}$ and his capital gains/losses as $X_{3,i}$. Let E_t^i denote the investor's expectation operator at time t . His time t utility, U_t^i , is then given by

$$U_t^i = E_t^i[v(X_{3,i})], \quad (3)$$

¹⁹In reality, an investor's one-period-ahead dividend forecasts might be correlated. As a robustness check, we also try the following specification to capture this correlation: $q_{i,t+1} = \rho q_{i,t} + (1 - \rho)\varepsilon_{i,t+1}$ with $\rho \in (0,1)$, where $q_{i,t}$ follows a beta distribution and $\varepsilon_{i,t+1}$ follows a uniform distribution. If $\rho = 0$, then we return to the specification in the main text in which his forecasts are independent over time; if $\rho = 1$, then an investor's forecasts about dividend growth rate are constant over time.

²⁰We also considered a model, similar to Barberis *et al.* (2001), in which an investor derives two kinds of utilities --- the standard consumption utility and prospect theory utility --- and obtained similar results.

where

$$X_{3,i} = W_{3,i} - R_f^2 W_{1,i}, \quad (4)$$

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\alpha & \text{if } x < 0 \end{cases}, \quad (5)$$

with $0 < \alpha \leq 1$ and $\lambda \geq 1$.

Here, the function $v(\cdot)$ is the standard value function of prospect theory proposed by Tversky and Kahneman (1992). The argument of $v(\cdot)$ is the capital gain/loss, $X_{3,i}$, not the final period wealth, $W_{3,i}$. Function $v(\cdot)$ is concave for gains and convex for losses, meaning that investors are risk averse in the domain of gains and risk-seeking in the domain of losses; it has a kink at the origin, implying a greater sensitivity to losses than to gains of the same magnitude. Parameter α governs its concavity/convexity and parameter λ controls loss aversion. For simplicity, we don't explore prospect theory's probability weighting feature in the above preference specification and just apply the standard expectation operator E_t^i . The primary effect of probability weighting is to overweight small probabilities; it therefore has its biggest impact on securities with highly skewed returns. Since most stocks are not highly skewed, we do not expect probability weighting to be central to the link between prospect theory and the disposition effect. Indeed, Hens and Vlcek (2006) find that probability weighting only plays a minor role in determining whether prospect theory predicts the disposition effect.

In equation (4), we follow the literature (e.g., Gomes, 2005; Barberis and Huang, 2008; Barberis and Xiong, 2009) and define the capital gain/loss as $X_{3,i} = W_{3,i} - R_f^2 W_{1,i}$.²¹ That is, we take a reference point as an investor's final wealth which he could have earned by investing in bonds when he was young and middle-aged.²² The gain/loss from a particular stock sale is calculated as the difference between the reference point and the investor's final wealth resulting from buying and selling this stock. For example, if investor i buys a stock at price P^B at age 1, sells it at price P^S and collects a dividend $D_{2,i}$ at age 2, and he then reinvests $P^S + D_{2,i}$ in bonds, getting back $R_f P^S + R_f D_{2,i}$ at age 3. If he had not bought the stock at age 1, but had invested P^B in bonds and held them till age 3, then he would have collected $R_f^2 P^B$ at age 3. Therefore, the gain/loss from this stock sale is $X_{3,i} = R_f P^S + R_f D_{2,i} - R_f^2 P^B$. This definition reflects the idea that an investor usually starts considering the stock investment as a loss if he could have earned more from investing in the riskless bond.

2.2.4 Timeline

To summarize, in the model, the exogenous random variables are θ_t and $q_{i,t}$,

²¹Two implementations of prospect theory have been proposed in the literature. The first implementation defines prospect theory over *annual* gains/losses (Benartzi and Thaler, 1995; Barberis *et al.*, 2001; Barberis and Huang, 2008). Another implementation is to define prospect theory over "*realized* gains/losses as in Barberis and Xiong (2008, 2009). The two implementations will be identical in our setup, because investors are allowed to hold only one unit of stock over their lifetimes.

²²In Appendix 2.A.1, we further show that our results are robust to taking purchase prices as reference points.

and the exogenous parameters of the model are $\theta_H > 0$, $\theta_L > 0$, $R_f > 1$, $0 < \alpha \leq 1$ and $\lambda \geq 1$. The order of events in each period t is shown in Figure 2.1. At the beginning of period t , age-1 investors are born and receive consumption good endowments. The dividend growth rate θ_t is realized, and all investors observe θ_t . The idiosyncratic belief shock $q_{i,t}$ is realized, and investor i observes $q_{i,t}$. All investors trade in the stock and bond market; age-2 and age-3 investors carry stocks to the market; after trading, age-1 and age-2 investors hold stocks. At the end of period t , age-3 investors receive prospect theory utility and exit the economy.

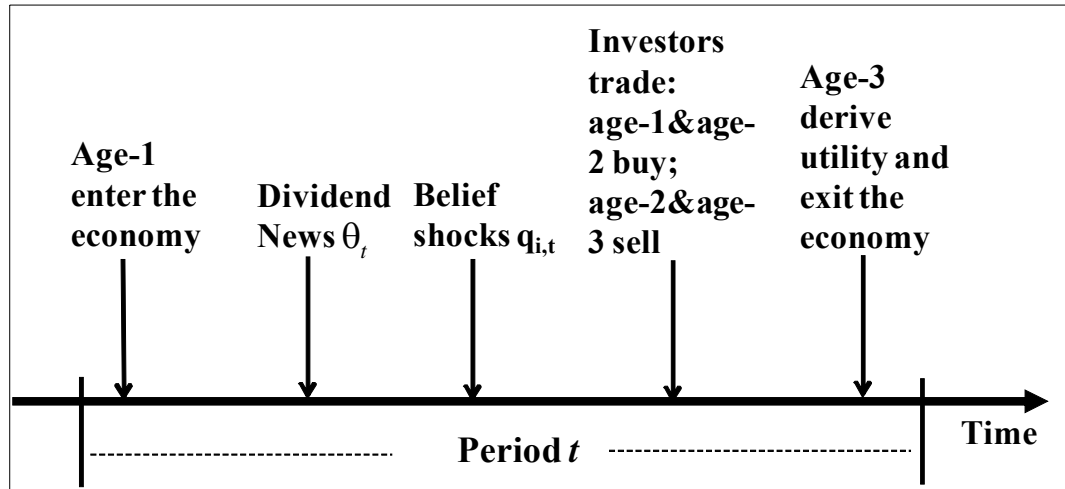


Figure 2.1 Timeline

Figure 2.1 plots the order of events in period t .

Our OLG setup can be understood as a stylized way of describing how different types of investors existing in real markets interact with each other. Our model economy can be linked to reality as follows. The potential buyers, namely an age-1 investor and an age-2 investor without a stock, correspond

respectively to a new participant and to a wait-and-seer who has been sitting in the market for some time. The potential sellers, namely an age-3 investor and an age-2 investor owning a stock, correspond respectively to a pure noise investor, one who has no discretion with regard to the timing of his trade, and to a discretionary liquidity investor, one who can determine when to trade.²³

2.2.5 Extension: A Multi-Stock Setting

So far, we have assumed just one risky asset, but our analysis has implications for the cross-section property of stock returns, so long as the investor engages in mental accounting or narrow framing (Thaler, 1980, 1985), thus deriving prospect theory utility separately from the trading profit on each distinct stock. This assumption is always present in the literature relating prospect theory to the disposition effect (e.g., Odean, 1998; Barberis and Xiong, 2009). Kumar and Lim (2008) also document that narrow framers indeed exhibit more of a disposition effect. Formally, we can consider an economy with N stocks, in which each stock has *i.i.d.* dividend processes with distribution given by equation (1), investors hold heterogeneous beliefs about the dividend growth rates and experience belief changes in their lifetimes, and these investors derive prospect theory utility from accumulative trading profits at the level of individual stocks. Then we can still use the conditions that characterize the equilibrium in the single stock setting --- more precisely, equations (6) through (23) --- to define an equilibrium, stock by stock, in this

²³The importance of differentiating a pure noise investor from a discretionary liquidity investor has been emphasized in the microstructure literature, for example, Admati and Pfleiderer (1988).

multi-stock setting. In Section 2.4, we conduct such an analysis and calculate the average returns to the winners-minus-losers portfolio to examine whether price momentum exists in our model economies.

2.3 Equilibrium

We now derive equilibrium asset prices. Let $f_t = \frac{P_t}{D_t}$ denote the price-dividend ratio in period t . To ease exposition, the investors of age 2 who *have* (*don't have*) a stock when they enter the market are referred to as *age-2-1 investors* (*age-2-0 investors*). Let z_t be the mass of age-2-1 investors in period t , i.e., z_t captures the distribution of stocks. Then in period t , the state of the economy is $S_t = (\theta_t, f_{t-1}, z_t)$. In equilibrium, the stock price-dividend ratios will be a function of the state vector, $f_t = f(S_t)$. The three variables θ_t , f_{t-1} and z_t affect stock prices because (i) θ_t and f_{t-1} affect age-2-1's investment decisions through the disposition effect, and (ii) z_t relates to aggregate effect on prices of age-2-1 investors as a whole. We construct the price-dividend function f by solving investors' optimal decisions backwards and using the market clearing condition.

2.3.1 Age-3 Investors' Decisions

A typical investor i of age 3 faces a state vector $(S_t, q_{i,t})$. His decision is simple: if he has a stock, he sells it and derives prospect theory utility from his trading profit; if he does not have a stock, he just waits until the end of the period and receives prospect theory utility. In sum, age-3 investors will sell $1 - z_t$ stocks as a whole.

2.3.2 Age-2 Investors' Decisions

A typical investor i of age 2 faces a state vector $(S_t, q_{i,t}, h_{i,t-1})$, where $h_{i,t-1} = 1$ if he belongs to age-2-1 and $h_{i,t-1} = 0$ if he belongs to age-2-0. An age-2-1 investor decides whether to sell the stock, and an age-2-0 investor decides whether to buy a stock.

Let us first look at the age-2-1 investors. If an age-2-1 investor continues to hold the stock, what is his expected prospect theory utility? In the next period, he will sell the stock at price P_{t+1} , resulting in a gain/loss

$$P_{t+1} + D_{t+1} + R_f D_t - R_f^2 P_{t-1} = G_{1 \rightarrow 1}^{t+1} D_{t-1}, \quad (6)$$

with $G_{1 \rightarrow 1}^{t+1} = (f_{t+1} + 1)\theta_t \theta_{t+1} + R_f \theta_t - R_f^2 f_{t-1}$.

As a result, his expected utility is

$$U_{1 \rightarrow 1}(S_t, q_{i,t}) = E_t^i[v(G_{1 \rightarrow 1}^{t+1})] D_{t-1}^\alpha, \quad (7)$$

where E_t^i is the subjective expectation operator conditional on investor i 's period t information set $\mathbf{F}_t^i = \{S_t, q_{i,t}\}$. Here, investor i takes expectation over the random variables θ_{t+1} and f_{t+1} according to his subjective belief [equation (2)] and the transition law of the state vector [equation (22)].

If he sells the stock, what is his expected prospect theory utility? Since he sells at price P_t , then his gain/loss is

$$R_f P_t + R_f D_t - R_f^2 P_{t-1} = G_{1 \rightarrow 0}^t D_{t-1}, \quad (8)$$

with $G_{1 \rightarrow 0}^t = (f_t + 1)R_f \theta_t - R_f^2 f_{t-1}$.

Therefore, his expected utility is

$$U_{1 \rightarrow 0}(S_t) = v(G_{1 \rightarrow 0}^t) D_{t-1}^\alpha. \quad (9)$$

If $U_{1 \rightarrow 1}(S_t, q_{i,t}) \geq U_{1 \rightarrow 0}(S_t)$, then investor i will continue to hold the stock. That is, those with sufficiently large belief shocks $q_{i,t}$ will not sell their stocks.

To sum up, the optimal decision of an age-2-1 investor is

$$h(S_t, q_{i,t}, 1) = 1_{U_{1 \rightarrow 1}(S_t, q_{i,t}) \geq U_{1 \rightarrow 0}(S_t)} = 1_{E_t^i[v(G_{1 \rightarrow 1}^{t+1})] \geq v(G_{1 \rightarrow 0}^t)}. \quad (10)$$

The corresponding *indirect* value function is²⁴

$$\begin{aligned} V(S_t, q_{i,t}, 1) &= \hat{V}(S_t, q_{i,t}, 1) D_{t-1}^\alpha, \\ \text{with } \hat{V}(S_t, q_{i,t}, 1) &= h(S_t, q_{i,t}, 1) E_t^i[v(G_{1 \rightarrow 1}^{t+1})] + [1 - h(S_t, q_{i,t}, 1)] v(G_{1 \rightarrow 0}^t). \end{aligned} \quad (11)$$

After trading, the fraction of those age-2-1 investors who continue to hold on to their stocks is

$$H_2(S_t, 1) = \int_0^{z_t} h(S_t, q_{i,t}, 1) di = z_t E[h(S_t, q_{i,t}, 1) | S_t] \quad (12)$$

where the second equality follows from the law of large numbers and the

²⁴Note that the *indirect* value function, $V(S_t, q_{i,t}, 1)$, is different from the value function of prospect theory $v(\cdot)$. Function $v(\cdot)$ corresponds to a standard Bernoulli utility function in the choice theory under uncertainty, but function $V(S_t, q_{i,t}, 1)$ is the indirect utility function which has taken into account the investor's optimal decisions.

expectation is taken over the random variable $q_{i,t}$, which follows a uniform distribution over $[0,1]$.

Next, let us check the age-2-0 investors. If an age-2-0 investor decides to buy a stock, then he will have a gain/loss

$$\begin{aligned} P_{t+1} + D_{t+1} - R_f P_t &= G_{0 \rightarrow 1}^{t+1} D_{t-1}, \\ \text{with } G_{0 \rightarrow 1}^{t+1} &= (f_{t+1} + 1)\theta_t \theta_{t+1} - R_f f_t \theta_t, \end{aligned} \quad (13)$$

and have expected prospect theory utility

$$U_{0 \rightarrow 1}(S_t, q_{i,t}) = E_t^i[v(G_{0 \rightarrow 1}^{t+1})] D_{t-1}^\alpha. \quad (14)$$

If he decides not to buy a stock, then his utility is 0. So an age-2-0 investor's optimal decision is

$$h(S_t, q_{i,t}, 0) = 1_{U_{0 \rightarrow 1}(S_t, q_{i,t}) \geq 0} = 1_{E_t^i[v(G_{0 \rightarrow 1}^{t+1})] \geq 0}, \quad (15)$$

and the corresponding *indirect* value function is

$$V(S_t, q_{i,t}, 0) = \hat{V}(S_t, q_{i,t}, 0) D_{t-1}^\alpha, \text{ with } \hat{V}(S_t, q_{i,t}, 0) = h(S_t, q_{i,t}, 0) E_t^i[v(G_{0 \rightarrow 1}^{t+1})] \quad (16)$$

After trading, the aggregate stock holding of age-2-0 investors is

$$H_2(S_t, 0) = \int_0^{1-z_t} h(S_t, q_{i,t}, 0) di = (1 - z_t) E[h(S_t, q_{i,t}, 0) | S_t] \quad (17)$$

2.3.3 Age-1 Investors' Decisions

A typical investor i of age 1 faces a state vector $(S_t, q_{i,t})$. If he decides to buy a stock, then his expected prospect theory utility is

$$U_1(S_t, q_{i,t}) = E_t^i[V(S_{t+1}, q_{i,t+1}, 1)] = \hat{U}_1(S_t, q_{i,t}) D_t^\alpha, \quad (18)$$

with $\hat{U}_1(S_t, q_{i,t}) = E_t^i[\hat{V}(S_{t+1}, q_{i,t+1}, 1)]$

and if he decides not to buy a stock, then his expected utility is

$$U_0(S_t, q_{i,t}) = E_t^i[V(S_{t+1}, q_{i,t+1}, 0)] = \hat{U}_0(S_t, q_{i,t}) D_t^\alpha, \quad (19)$$

with $\hat{U}_0(S_t, q_{i,t}) = E_t^i[\hat{V}(S_{t+1}, q_{i,t+1}, 0)]$.

Therefore, his optimal decision is

$$h(S_t, q_{i,t}) = 1_{\hat{U}_1(S_t, q_{i,t}) \geq \hat{U}_0(S_t, q_{i,t})} \quad (20)$$

So after trade, age 1 as a whole will hold

$$H_1(S_t) = \int_0^1 h(S_t, q_{i,t}) di = E[h(S_t, q_{i,t}) | S_t] \quad (21)$$

2.3.4 Evolution of State Variables

The state vector S_t evolves according to the following equation

$$S_{t+1} = (\theta_{t+1}, f_t, z_{t+1}) = (\theta_{t+1}, f(S_t), H_1(S_t)), \quad (22)$$

where functions $H_1(S_t)$ [given by (21)] and $f(S_t)$ are both endogenously

determined. The random process $\{\theta_{t+1}\}_{t=1}^{\infty}$ is *i.i.d.* with distribution $\Pr(\theta_{t+1} = \theta_H) = \Pr(\theta_{t+1} = \theta_L) = \frac{1}{2}$ [i.e., equation (1)]. When investors make decisions, however, they believe that θ_{t+1} evolves according to $\Pr_t^i(\theta_{t+1} = \theta_H) = q_{i,t}$ [i.e., equation (2)]. Since S_t is in the investors' information set, they know the other two variables in S_{t+1} , i.e., f_t and z_{t+1} .

2.3.5 Market Clearing Condition

The market clearing condition is

$$H_1(S_t) + H_2(S_t, 0) + H_2(S_t, 1) = 1, \quad (23)$$

which states that the stock holdings from age-1, age-2-0, and age-2-1 add up to the total stock supply 1. An equilibrium price-dividend function f is implicitly determined by equations (6) through (23).

We adopt the equilibrium concept of Radner (1972), known as equilibrium of plans, prices, and price expectations. An equilibrium is formally defined as follows.

Definition. An equilibrium consists of decision rules, $h(S_t, q_{i,t})$, $h(S_t, q_{i,t}, 0)$ and $h(S_t, q_{i,t}, 1)$, and a law of motion $S_{t+1} = (\theta_{t+1}, f_t, z_{t+1}) = (\theta_{t+1}, f(S_t), H_1(S_t))$ such that

(1) the decision rules maximize investors' expected prospect theory utility conditional on their information;

(2) markets clear: $H_1(S_t) + H_2(S_t, 0) + H_2(S_t, 1) = 1$ for almost every realization of S_t ; and

(3) *the law of motion is generated by decision rules.*

Note that the above definition of equilibrium has implicitly incorporated prices into the price-dividend ratio function in the law of motion.

2.3.6 Benchmark Case: Standard Risk Neutral Utility

Suppose $\alpha = \lambda = 1$. Concavity/convexity and loss aversion, two distinctive features of prospect theory, will vanish, reducing the preferences to a standard risk neutral utility representation. This works as a benchmark economy to illustrate that all our results are driven by prospect theory preferences. We don't use a standard, risk averse preference, such as power utility functions, as the benchmark, because risk aversion *per se* can *qualitatively* generate a disposition effect through portfolio rebalancing, although Odean (1998) argues that portfolio rebalancing cannot *quantitatively* account for the disposition effect.²⁵ Risk neutrality removes this contamination and therefore gives cleaner results.

When investors are risk neutral, i.e., when $\alpha = \lambda = 1$, both the price-dividend ratio and the mass of age 2-1 investors are constant:

$f_t \equiv f = \frac{E(\theta_{t+1})}{R_f - E(\theta_{t+1})} = \frac{\frac{1}{2}\theta_H + \frac{1}{2}\theta_L}{R_f - (\frac{1}{2}\theta_H + \frac{1}{2}\theta_L)}$ and $z_t = \frac{1}{2}$. This result can be obtained by examining equations (6) to (23).

²⁵If investors sell winners due to portfolio rebalancing, then they will partially reduce their position in a winning stock, rather than sell the entire position of the stock. Odean (1998) shows that the disposition effect still remains strong, even when the sample is restricted to transactions of investors' entire holdings of a stock, i.e., to those transactions not motivated by portfolio rebalancing. This suggests that portfolio rebalancing cannot entirely account for the disposition effect.

In fact, the constant price-dividend ratio is consistent with the simple Gordon rule: $P_t = \frac{E(\theta_{t+1})D_t}{R_f - E(\theta_{t+1})}$. Intuitively, the potential buyers of stocks are those age-1 and age-2 investors who hold optimistic views about next period's dividend realization; the marginal buyer's subjective belief, coinciding with the true distribution of the dividend process, brings the stock price equal to the sum of the discounted expected dividends. In this special case, we have an *i.i.d.* return process,

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{f+1}{f} \theta_{t+1},$$

with mean equal to R_f . The age-2-1 investors no longer exhibit a disposition effect, because half of them, those who have received pessimistic belief shocks (i.e., $q_{i,t} < 1/2$), will always liquidate stocks no matter whether they face gains or losses.

For the general cases of $\alpha < 1$ or $\lambda > 1$, we have to numerically solve the price-dividend function $f(\cdot, \cdot, \cdot)$ and age-1 investors' stock demand function $H_1(\cdot, \cdot, \cdot)$. The basic methodology is as follows: starting from an initial conjecture of $f^{(0)}(\cdot, \cdot, \cdot)$ and $H_1^{(0)}(\cdot, \cdot, \cdot)$, solve $f^{(1)}(S_t)$ and $H_1^{(1)}(S_t)$ on a grid of S_t from equations (6)-(23), and continue this process until $f^{(n)}(\cdot, \cdot, \cdot) \rightarrow f(\cdot, \cdot, \cdot)$ and $H_1^{(n)}(\cdot, \cdot, \cdot) \rightarrow H_1(\cdot, \cdot, \cdot)$.

2.4 Numerical Results and Intuitions

In this section we solve equations (6) through (23) for the two endogenous functions in the law of motion: the price-dividend ratio function, $f(\cdot, \cdot, \cdot)$, and the

aggregate demand function of age-1 investors, $H_1(\cdot, \cdot)$. We then use simulations to show that the two components of prospect theory, diminishing sensitivity and loss aversion, make exactly opposite predictions regarding individual trading behavior, return predictability, and the correlation between returns and volume. Specifically, Subsection 2.4.2 demonstrates that diminishing sensitivity drives a disposition effect, which in turn leads to momentum in the cross-section of stock returns and a positive correlation between returns and volume. Subsection 2.4.3 shows, on the other hand, that loss aversion predicts a reversed disposition effect and reversal in the cross-section of stock returns, as well as a negative correlation between returns and volume. Subsection 2.4.4 conducts further quantitative analysis to examine how successful prospect theory is in explaining price momentum, and suggests testable empirical predictions.

2.4.1 Calibrating Technology Parameters

There are five exogenous parameters in our model: two preference parameters (λ and α) and three technology parameters (θ_H , θ_L and R_f). Since we are interested in the implications of preferences, we allow the preference parameters to vary over a certain range. But we calibrate the technology parameters as follows. We take one period to be one year, and thus set the net risk-free rate to $R_f - 1 = 3.86$ percent, a choice adopted by Barberis and Huang (2001). Since the disposition effect refers to the behavior of individual stocks, we choose dividend parameters to match the mean and standard deviation of the dividend growth rate of a typical individual stock. Barberis and Huang (2001) estimate the moments of individual stock dividend growth using the COMPUSTAT database, and based on their results, we set

$\theta_H = 1.28$ and $\theta_L = 0.76$, such that the mean and volatility of the net growth rate of the dividend are 2.24% and 25.97%, respectively. Table 2.1 summarizes our choice of technology parameters.

Table 2.1 Technology Parameter Values

We take one period to be one year. Dividend parameters (θ_H and θ_L) are calibrated to generate a dividend growth rate with the mean and standard deviation equal to 2.24% and 25.97%, respectively.

Parameter	Values
Risk-free rate	
R_f	1.0386
Dividend parameters	
θ_H	1.2821
θ_L	0.7628

2.4.2 Implications of Diminishing Sensitivity

We obtain the implications of diminishing sensitivity through comparative static analysis with respect to parameter λ , which governs the curvature of the value function. To ensure that our results are completely driven by the concavity/convexity component of prospect theory, in this subsection we also set parameter λ at 1 to remove the loss aversion feature of the preference. Table 2.2 presents the main results for a range of values of α : 0.2, 0.5, 0.88 and 1. In particular, when $\alpha = 1$, the investor is risk neutral, which provides a benchmark for highlighting the fact that our results stem from prospect theory preferences. The value of 0.88 is the number estimated by Tversky and Kahneman (1992). Our results demonstrate that, in a general equilibrium

setting, diminishing sensitivity drives the disposition effect, the momentum effect and the comovement between returns and volume. We also find that diminishing sensitivity alone, in the absence of loss aversion, raises equity premiums.

2.4.2.1 Disposition Effects

We use the following measure to test whether our model can generate a disposition effect,

$$DispEffect = \frac{E\left[\frac{z_t - H_2(S_t, 1)}{z_t} \middle| G_{1 \rightarrow 0}^t > 0\right]}{E\left[\frac{z_t - H_2(S_t, 1)}{z_t} \middle| G_{1 \rightarrow 0}^t < 0\right]}. \quad (24)$$

If $DispEffect > 1$, then we conclude that investors exhibit the disposition effect in our model. The numerator of $DispEffect$ is the average fraction of age-2-1 investors who close their positions facing a capital gain. This term is the theoretical analog to Odean's (1998) proportion of gains realized (PGR), i.e., the number of gains that are realized as a fraction of the total number of gains that could have been realized. Similarly, the denominator of $DispEffect$ is the average fraction of age-2-1 investors who realize losses and corresponds to Odean's proportion of losses realized (PLR). Odean uses the difference between PGR and PLR to measure the disposition effect. In equation (24), we instead adopt a ratio of PGR to PLR to remove the effect of equity premiums on the magnitudes of PGR or PLR.²⁶

²⁶Brown *et al.* (2006) also use the ratio of PGR to PLR to measure the disposition effect when examining Australian stock trading data.

Table 2.2 Implications of Diminishing Sensitivity

PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$, and if $DispEffect > 1$, then a disposition effect exists. $MomEffect = E(R_{t+1} | \theta_t = \theta_H) - E(R_{t+1} | \theta_t = \theta_L)$. WML is the simulated average momentum portfolio return in the multi-stock setting. If $MomEffect > 0$ and $WML > 0$, then a momentum effect exists. $Q_t = 1 - H_2(S_t, 1)$ is the turnover, or aggregate selling, in period t . Technology parameter values are fixed at the values in Table 2.1: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. The preference parameter $\lambda \geq 1$ determines loss aversion; in this table, we deliberately set λ as 1, so that the investor is not averse to loss.

	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.88$	$\alpha = 1$
(i) Disposition Effect				
PGR	0.49	0.56	0.53	0.50
PLR	0.29	0.36	0.47	0.50
<i>DispEffect</i>	1.73	1.56	1.12	1.00
(ii) Momentum Effect				
$E(R_{t+1} \theta_t = \theta_H)$	1.1295	1.0934	1.0516	1.0386
$E(R_{t+1} \theta_t = \theta_L)$	1.0158	1.0439	1.0410	1.0386
<i>MomEffect</i>	11.37%	4.95%	1.06%	0.00%
<i>WML</i>	10.91%	4.67%	1.06%	0.00%
(iii) Turnover				
$Corr(R_t, Q_t)$	0.52	0.83	0.92	0.00
(iv) Equity Premium				
$E(R_t - R_f)$	3.43%	3.01%	0.82%	0.00%
$E[H_1(S_t)]$	0.43	0.46	0.49	0.50

To obtain the two conditional moments in equation (24), we simulate a long time series $\{\theta_t\}_{t=1}^{\infty}$ of 500,000 independent draws from the distribution described in equation (1). Then we use the solved functions $f(\cdot, \cdot, \cdot)$ and $H_1(\cdot, \cdot, \cdot)$ to calculate f_t and z_{t+1} and get the time series $\{S_t\}_{t=1}^{\infty}$. When we do this, we

also compute $\{H_2(S_t, 1)\}_{t=1}^{\infty}$ and $\{G_{1 \rightarrow 0}^t\}_{t=1}^{\infty}$ along the way, using equations (H21) and (G10). We compute sample moments from these simulated data to serve as approximations of population moments.

Table 2.2 reports the results for different values of α . The case of $\alpha = 1$ corresponds to a linear value function, when investors don't exhibit a disposition effect, so that $DispEffect = 1$. As we gradually decrease α from 1 to 0.2, the value function becomes more curved along the way, and the value of $DispEffect$ increases monotonically from 1 to 1.73, giving rise to an even stronger disposition effect. The mechanism behind this result is exactly Odean's (1998) intuition: risk aversion (risk-seeking) for gains (losses) causes an age-2-1 investor more (less) likely to sell the stock.

Figure 2.2 graphs this intuition for the case of $\alpha = 0.5$. Here, from the simulated time series of state vectors, we randomly choose a realization of $(f_{t-1}, z_t) = (20.01, 0.50)$, and then graph the possible gains/losses together with the associated prospect theory utilities faced by an age-2-1 investor in periods t and $t+1$.²⁷ The period t gains/losses as well as prospect theory utilities from liquidating the stock [i.e., $(G_{1 \rightarrow 0}^t, v(G_{1 \rightarrow 0}^t))$] are marked with dots, while the period $t+1$ gains/losses and prospect theory utilities from keeping the stock [i.e., $(G_{1 \rightarrow 1}^{t+1}, v(G_{1 \rightarrow 1}^{t+1}))$] are marked with circles.

Good dividend news ($\theta_t = \theta_H$) will bring an age-2-1 investor to the point of

²⁷The result is robust to the choice of (f_{t-1}, z_t) .

choosing a sure medium gain (6.5, Point H in the figure) versus a gamble which offers either a smaller gain (1.18, Point HL) or a larger gain (14.45, Point HH) with some probabilities. Whether an age-2-1 investor will continue to hold the stock depends on his one-period-ahead dividend forecast. In this example, those age-2-1 investors who believe, with probability higher than 0.54 (i.e., $\frac{v(6.5)-v(1.18)}{v(14.45)-v(1.18)}$), that the next period dividend growth rate (θ_{t+1}) will take a high value (θ_H) will continue to hold the risky stock.

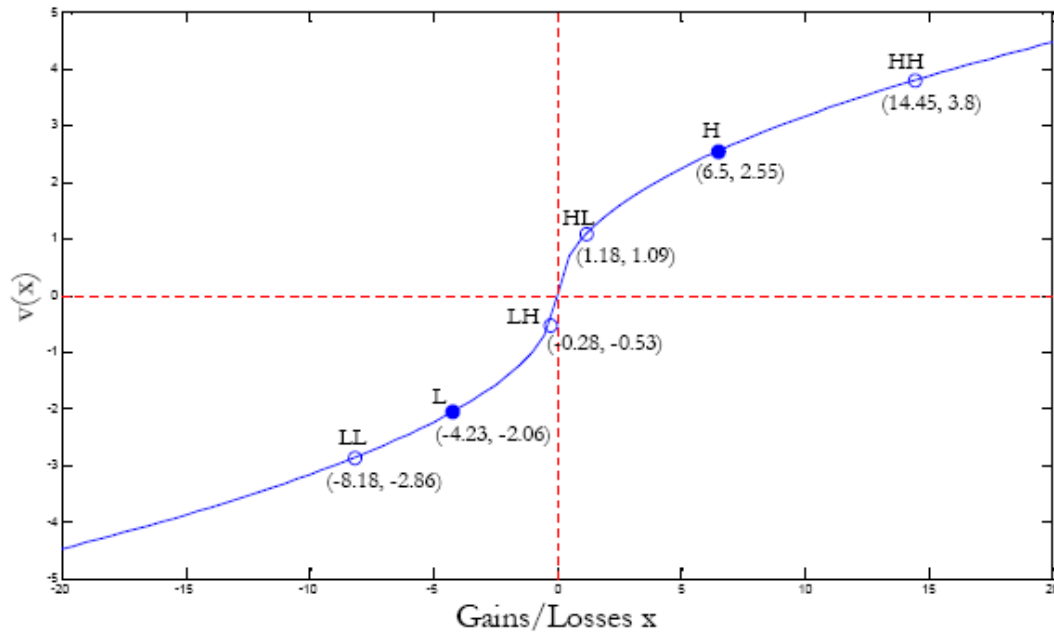


Figure 2.2 Diminishing Sensitivity Drives the Disposition Effect

Figure 2.2 graphs the possible capital gains/losses, as well as prospect theory utilities, faced by an age-2-1 investor. If this investor liquidates his stock, his capital gains/losses, together with his prospect theory utilities, are marked with dots; if he keeps the stock, then his possible future capital gains/losses and his prospect theory utilities are marked with circles. The two endogenous state variables are $f_{t-1} = 20.01$ and $z_t = 0.50$. The parameter values are $\theta_H = 1.2821$, $\theta_L = 0.7628$, $R_f = 1.0386$, $\lambda = 1$ and $\alpha = 0.5$.

What will happen if the dividend news is negative ($\theta_t = \theta_L$) at period t ? If an age-2-1 investor sells the stock, he experiences a sure loss (-4.23, Point L); if

he continues to hold the stock, he faces the gamble of a smaller loss (-0.28 , Point LH) or an even larger loss (-8.18 , Point LL). In this example, those age-2-1 investors who believe that $\theta_{t+1} = \theta_H$ with probability lower than 0.35 (i.e., $\frac{v(-4.23)-v(-8.18)}{v(-0.28)-v(-8.18)}$), will liquidate their stocks. Note that the cutoff probability in the low dividend realization case, 0.35 , is lower than that in the high dividend realization, 0.54 . This precisely supports the informal argument, which relies on prospect theory to explain the disposition effect: the investor's belief about expected return must fall further to motivate the sale of a stock that has already declined than one that has appreciated (Odean, 1998, p. 1777).

Table 2.2 suggests that PGR and PLR respond to a change in α differently: as α falls from 1 to 0.2, PGR first goes up from 0.50 to 0.56 and then goes down to 0.49, while PLR continuously decreases from 0.50 to 0.29. There are two forces at work here. As α becomes smaller, the value function is more concave for gains and more convex for losses, causing the investor to be more likely to sell winners and hold losers, and hence generating a higher PGR and a lower PLR. However, as α falls, the expected stock return rises and the stock becomes more attractive to the investor, which will be discussed shortly; this decreases the investor's propensity to sell the stock no matter whether he is facing gains or losses, and therefore leads to both a lower PGR and a lower PLR. In sum, as α decreases, both forces tend to lower PLR, while the first force tends to raise PGR and the second to lower PGR. As α falls slightly below 1, the first force dominates, and we observe a higher PGR, but once α falls sufficiently, the second force catches up and we obtain a lower PGR.

2.4.2.2 Momentum

Following Barberis *et al.* (1998), who also rely on a model with one risky asset to explain the cross-section of stock returns, we measure momentum as

$$MomEffect = E(R_{t+1} | \theta_t = \theta_H) - E(R_{t+1} | \theta_t = \theta_L), \quad (25)$$

i.e., the difference in the expected return following a positive shock and following a negative shock. If $MomEffect > 0$, then we claim that there is momentum in the stock returns. The two moments in equation (25) are obtained using simulations. The results are also reported in Table 2.2. Since $MomEffect > 0$ for $\alpha < 1$, our model shows that the concavity/convexity feature of prospect theory preferences generates momentum in stock returns. Moreover, the momentum effect becomes stronger as we increase the curvature of the value function, i.e., decrease the value of α . For example, $MomEffect$ increases from 1.06% to 11.37% as α decreases from 0.88 to 0.2. The underlying reason for this momentum effect is simple. Following a positive shock ($\theta_t = \theta_H$), stock prices will rise, moving age-2-1 investors into their capital gain domain. Due to the concavity of the value function of prospect theory in the gain region, age-2-1 investors tend to close their stock positions, which depresses the stock price, generating higher subsequent returns. On the other hand, a negative shock ($\theta_t = \theta_L$) will decrease the stock price, driving age-2-1 investors into their capital loss domain. Convexity in the region of losses means that they are less likely to sell the stock absent a price premium; the stock price is therefore initially inflated, generating lower subsequent returns.

We also conduct a cross-section analysis and replicate the momentum effect in the empirical literature (e.g., Jegadeesh and Titman, 1993; Liu and Zhang, 2008). As discussed in the end of Section 2.2, we can extend our model to an economy with N stocks. We simulate dividend data on $N = 2,000$ independent stocks over $T = 10,000$ time periods, and then compute the resulting equilibrium return sequence for each stock. We create the winners-minus-losers zero cost portfolios as follows. In each period, we sort stocks into two equal-sized groups based on their last period returns and record the equal-weighted return of each group over the next period; in particular, R_t^{winner} (R_t^{loser}) is the return on the portfolio containing stocks with better (worse) performance. Repeating this each period produces long time series of returns on the winner and loser portfolios, namely $\{R_t^{winner}\}_{t=1}^T$ and $\{R_t^{loser}\}_{t=1}^T$. Our second measure of momentum is the difference in the average returns on these two portfolios:

$$WML = \frac{1}{T} \sum_{t=1}^T (R_t^{winner} - R_t^{loser}) \quad (26)$$

Table 2.2 also reports the results for this alternative measure. We find that the two measures for momentum are almost identical, so that they behave in precisely the same way: both *MomEffect* and *WML* are greater than 0 for $\alpha < 1$, and both decrease with α .

2.4.2.3 Turnover

Empirical studies show that there is more trading in rising markets than in falling markets (Statman *et al.*, 2006; Griffin *et al.*, 2007). In our model, the age-2-1 investors have a much greater propensity to sell stocks facing good

news ($\theta_t = \theta_H$) than facing bad news ($\theta_t = \theta_L$). This will contribute to a positive correlation between turnover and stock returns. Let $Q_t = 1 - H_2(S_t, 1)$ be the turnover or aggregate selling in period t . In Table 2.2, we report the simulated correlations between stock returns and turnovers, $Corr(R_t, Q_t)$. Indeed, we have $Corr(R_t, Q_t) > 0$ so long as $\alpha < 1$. This demonstrates that diminishing sensitivity drives a positive correlation between returns and volume.

As we gradually decrease α from 0.88 to 0.2, $Corr(R_t, Q_t)$ decreases from 0.92 to 0.52. The stock distributions (z_t) and price-dividend ratios (f_{t-1}) combine to contribute to this relationship, but they work in different ways when α varies. When α is close to 1, both z_t and f_{t-1} are almost constant at their values in the benchmark economy (i.e., $\alpha = 1$), so that the state of the economy is captured only by dividend growth rates (θ_t). Since the disposition effect causes returns and turnovers to vary with θ_t in the same direction, there is an almost perfect correlation between returns and volume. On the other hand, as α gets close to 0, both z_t and f_{t-1} will change over time and influence trading behavior. However, returns and volumes respond to the variation in z_t and f_{t-1} in opposite ways. For example, a larger z_t implies that more stocks are held by age-2-1 investors and fewer by age-3 investors; after trading, all age-3 investors will have to close their positions, even though this is not the case for age-2-1 investors; as a result, stock selling (i.e., trading volumes Q_t) will decrease with z_t , but at the same time, the decreasing selling pressure causes stock returns R_t to rise with z_t . The variation in z_t and f_{t-1} will therefore attenuate the positive correlation between returns and volume generated by θ_t . As a result, for $0 < \alpha < 1$, a lower α implies a lower $Corr(R_t, Q_t)$.

2.4.2.4 Equity Premiums

Our model also demonstrates that the S-shaped value function of prospect theory can help explain the equity premium puzzle. Table 2.2 reports the simulated equity premiums, $E(R_t - R_f)$, as well as average stock purchases by young people, $E[H_1(S_t)]$. As α gets smaller, the curvature of the value function becomes larger, and equity premiums become higher. Note that the positive equity premium is not due to loss aversion, since we have set $\lambda = 1$ in this section. Notably, a low α is also associated with a low $E[H_1(S_t)]$, suggesting that equity premiums are driven by the behavior of young people. The young investor makes investment decisions by comparing the expected utility from buying the stock to that from not buying. These utility levels are determined by his belief $q_{i,t}$ (current belief about θ_{t+1}), and by how he evaluates his future reactions to $q_{i,t+1}$ (future belief about θ_{t+2}). Those who are extremely optimistic (pessimistic), i.e., those with extremely high (low) values of $q_{i,t}$, always buy (not buy) the stock. It is those who have intermediate values of $q_{i,t}$ that care more about their future reactions to $q_{i,t+1}$. It turns out that only high realizations of $q_{i,t+1}$ will matter, because there will be no extra benefit of holding a stock from middle-aged till old when $q_{i,t+1}$ is low. Only when $q_{i,t+1}$ is high will holding the stock from middle-aged till old bring an extra benefit: a young investor who buys a stock now will enjoy a further gain if he keeps the stock, and one who doesn't buy now will enjoy a new gain if he buys the stock when middle-aged.

How does this extra gain associated with high $q_{i,t+1}$ relate to the current purchasing decision and the value function's curvature α ? Not buying now

means that when evaluating this gain, the young investor will stay in the origin of the value function, where the marginal utility is the highest; the more curved the value function, the higher is this marginal utility. In contrast, if he buys now, he will be pushed away from the origin because this gain has to be appended to an existing gain or loss, namely the one generated by holding the stock from young until middle-aged. In this case, the marginal utility is much smaller compared to that in the origin; the more curved the value function, the smaller is this marginal utility.

To summarize, the higher the curvature of the value function, the less a young investor will value the potential gain associated with high realizations of $q_{i,t+1}$, and the less they want to buy now, thereby depressing stock prices and raising equity premiums.

2.4.3 Implications of Loss Aversion

To obtain the implications of loss aversion, we conduct comparative static analysis with respect to the parameter λ . In Table 2.3, we present the results for a variety of values of λ : 1, 2.25, 3 and 4. In particular, $\lambda = 1$ is still our benchmark economy when the investor is risk neutral. The value of $\lambda = 2.25$ is the number estimated by Tversky and Kahneman (1992). To guarantee that our results are solely due to the loss aversion component, we always set parameter $\alpha = 1$ to remove the curvature feature of the prospect theory value function. Table 2.3 demonstrates that loss aversion drives a reversed disposition effect and reversal in the cross-section of stock returns, as well as a negative correlation between returns and volume. In addition, Table 2.3 produces a well-known result in the asset pricing literature: loss aversion can

raise equity premiums, such as Benartzi and Thaler (1995) and Barberis *et al.* (2001).

2.4.3.1 Reversed Disposition Effects

Again, when investors are risk neutral, i.e., when $\lambda = 1$, they don't exhibit a disposition effect, so that $DispEffect = 1$. When investors are loss averse, i.e., when $\lambda > 1$, we obtain a reversed disposition effect, since $DispEffect < 1$.

Moreover, as we gradually increase λ from 1 to 4, investors become more loss averse, and the value of $DispEffect$ decreases monotonically from 1 to 0.83, giving rise to an even stronger reversed disposition effect.

What's the intuition behind this result? The mechanism works through a combination of two forces: one is the kink at the origin of the value function, which is a direct implication of loss aversion; the other is the positive equity premium, which is an indirect equilibrium implication of loss aversion preferences. Roughly speaking, when investors are close to (far from) the kink, they are reluctant (inclined) to take risk, and want to sell (keep) the stock; when the average stock returns are higher than the risk free rate, bad (good) dividend news will bring investors relatively close to (far from) the kink, so that they are more (less) likely to liquidate the stock.

Table 2.3 Implications of Loss Aversion

PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$, and if $DispEffect < 1$, then a reversed disposition effect exists. $MomEffect = E(R_{t+1} | \theta_t = \theta_H) - E(R_{t+1} | \theta_t = \theta_L)$. WML is the simulated average momentum portfolio return in the multi-stock setting. If $MomEffect < 0$ and $WML < 0$, then there is reversal in the cross-section of stock returns. $Q_t = 1 - H_2(S_t, 1)$ is the turnover, or aggregate selling, in period t . Technology parameter values are fixed at the values in Table 2.1: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. Preference parameter α controls the curvature of the value function. In this table, we deliberately set α to be 1, so that the value function is piecewise linear.

	$\lambda = 1$	$\lambda = 2.25$	$\lambda = 3$	$\lambda = 4$
(i) Disposition Effect				
PGR	0.50	0.39	0.35	0.31
PLR	0.50	0.41	0.39	0.38
<i>DispEffect</i>	1.00	0.97	0.91	0.83
(ii) Momentum Effect				
$E(R_{t+1} \theta_t = \theta_H)$	1.0386	1.1000	1.1255	1.1519
$E(R_{t+1} \theta_t = \theta_L)$	1.0386	1.1006	1.1311	1.1632
<i>MomEffect</i>	0.00%	-0.06%	-0.56%	-1.13%
<i>WML</i>	0.00%	-0.23%	-0.85%	-1.48%
(iii) Turnover				
$Corr(R_t, Q_t)$	0.00	-0.70	-0.91	-0.94
(iv) Equity Premium				
$E(R_t - R_f)$	0.00%	6.17%	8.97%	11.89%

Figure 2.3 conducts an exercise to confirm this intuition for the case of $\lambda = 4$. Now that we assumed $\alpha = 1$ to remove the curvature, the investor's value function becomes piecewise linear with a kink at the origin due to loss aversion. Similar to the exercise in Figure 2.2, we randomly choose a realization of (f_{t-1}, z_t) , which is $(7.75, 0.48)$ in this case, from the simulated time series of state vectors. We then graph an age-2-1 investor's period t gains/losses as well as prospect theory utilities from liquidating the stock [i.e. $(G_{1 \rightarrow 0}^t, v(G_{1 \rightarrow 0}^t))$] with dots, and the period $t + 1$ gains/losses and prospect theory utilities from keeping the stock [i.e. $(G_{1 \rightarrow 1}^{t+1}, v(G_{1 \rightarrow 1}^{t+1}))$] with circles.

Good dividend news ($\theta_t = \theta_H$) will bring the investor to Point H. Bad dividend news ($\theta_t = \theta_L$) will bring him to Point L, which is closer to the kink relative to Point H. That is, the investor is more cautious in holding stocks at Point L than at Point H. Specifically, at Point H, if the investor liquidates the stock, he will lock in a medium gain of 3.29; if he keeps the stock, when he becomes old he will arrive either at Point HH, enjoying a large gain of 7.35, or at Point HL, enjoying a small gain of 1.46. Since both Point HH and Point HL are in the gain domain, the investor's behavior at Point H can be described as risk neutral. Of course, whether an age-2-1 investor will indeed continue to hold the stock depends on his one-period-ahead dividend forecast. In this example, those age-2-1 investors who believe that $\theta_{t+1} = \theta_H$ with probability higher than 0.31 (i.e., $\frac{v(3.29) - v(1.46)}{v(7.35) - v(1.46)}$), will continue to hold the risky stock.

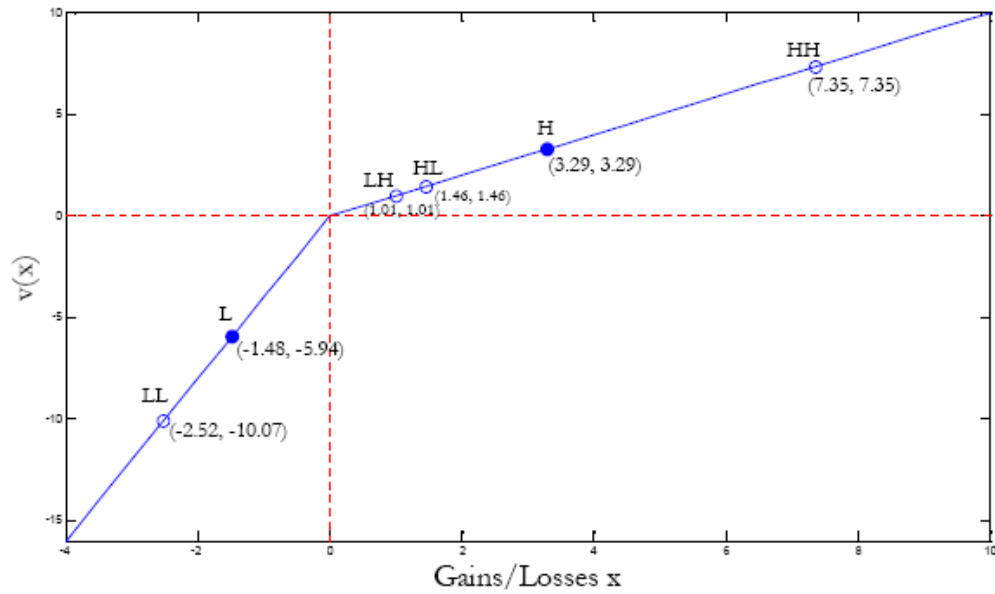


Figure 2.3 Loss Aversion Drives the Reversed Disposition Effect

Figure 2.3 graphs the possible capital gains/losses, as well as prospect theory utilities, faced by an age-2-1 investor. If this investor liquidates his stock, his capital gains/losses, together with his prospect theory utilities, are marked with dots; if he keeps the stock, then his possible future capital gains/losses and his prospect theory utilities are marked with circles. The two endogenous state variables are $f_{t-1} = 7.75$ and $z_t = 0.48$. The parameter values are $\theta_H = 1.2821$, $\theta_L = 0.7628$, $R_f = 1.0386$, $\lambda = 4$ and $\alpha = 1$.

At Point L, if the investor sells the stock, he will realize a loss of 1.48. If he keeps the stock, then he will arrive either at Point LH, enjoying a small gain of 1.01, or at Point LL, facing a large loss of 2.52. Because Point LH and Point LL straddle over the kink, the investor is reluctant to take a risk at Point L relative to Point H, at which point his behavior resembles risk neutrality. In this example, those age-2-1 investors who believe that $\theta_{t+1} = \theta_H$ with probability lower than 0.37 (i.e., $\frac{v(-1.48)-v(-2.52)}{v(1.01)-v(-2.52)}$), will liquidate their stocks.

In Table 2.3, we also observe that both PGR and PLR decrease with λ . This is because loss aversion raises equity premiums, making the investor less likely to sell stocks, whether facing good news or bad news. We also observe PGR decreases at a faster rate than PLR due to the reversed disposition effect.

2.4.3.2 Reversal

As discussed above, when $\lambda = 1$, the investor is risk neutral, and there is no momentum effect in the cross-section of stock returns, because both measures capturing momentum, *MomEffect* and *WML*, are equal to zero. But as long as $\lambda > 1$, i.e., as long as the investor is loss averse, we obtain reversal in the cross-section of stock returns, since both *MomEffect* and *WML* are negative. In particular, as we increase λ from 1 to 4, reversal gets stronger. This result demonstrates that the loss aversion feature of prospect theory has implications for return predictability.

The underlying reason for this result is similar to Grinblatt and Han's (2005). For example, facing good dividend news, age-2-1 investors are more likely to hold stocks according to the reversed disposition effect. This generates extra buying pressure, which will inflate stock prices and lead to lower stock returns later. Similarly, facing bad dividend news, those investors are likely to sell stocks and depress prices, generating higher subsequent returns.

2.4.3.3 Turnover

Table 2.3 also shows that loss aversion can generate a negative correlation between returns and volumes: $Corr(R_t, Q_t) < 0$ as long as $\lambda > 1$. This result is

also driven by trading by age-2-1 investors, who, due to the reversed disposition effect, have a much greater propensity to sell stocks in down markets ($\theta_t = \theta_L$) than in up markets ($\theta_t = \theta_H$), contributing to a negative correlation between turnover and stock returns.

As we gradually increase λ from 1 to 4, $Corr(R_t, Q_t)$ monotonically decreases from 0 to -0.94 . This pattern is different from the relationship between $Corr(R_t, Q_t)$ and α in Table 2.2 and can be understood as follows. In Table 2.2, when we vary α while fixing λ , dividend news θ_t contributes to a positive $Corr(R_t, Q_t)$ via the disposition effect, while the other endogenous state variables, stock distributions (z_t) and price-dividend ratios (f_{t-1}), tend to generate a negative $Corr(R_t, Q_t)$. These two forces are counteracting. On the other hand, in Table 2.3, when we vary λ and fix α , dividend news θ_t also leads to a negative $Corr(R_t, Q_t)$ through the reversed disposition effect, which strengthens the impact of the two endogenous state variables on $Corr(R_t, Q_t)$.

2.4.3.4 Equity Premiums

Table 2.3 also reproduces the well-known result that loss aversion can raise equity premiums (e.g., Benartzi and Thaler, 1995; Barberis *et al.*, 2001). As we increase λ from 1 to 4, equity premiums rise from 0 to 12%. This result is intuitive: loss aversion means that investors are more sensitive to losses than to gains, and since stocks often perform poorly and investors often face losses, a large premium is required to convince them to hold stocks. The asset pricing literature studying loss aversion has focused primarily on its implications for the equity premium, that is, the average *level* of stock returns. Our model, on the other hand, shows that loss aversion can lead to reversal in the cross-

section of stock returns, suggesting, in turn, that loss aversion may also be a useful ingredient for equilibrium models trying to understand return *predictability*.

2.4.4 Quantitative Analysis and Testable Predictions

In this Subsection, we conduct further quantitative analysis to examine how successful prospect theory is in explaining price momentum and derive testable empirical predictions which are either unique to our model or consistent with the existing empirical studies.

2.3.4.1 Quantitative Analysis: How Successful is Prospect Theory?

So far, we have shown that there are two counteracting forces in equilibrium -- - diminishing sensitivity and loss aversion --- driving the disposition effect, the momentum effect and the correlation between returns and volumes. In order to understand how successful prospect theory is in explaining price momentum, we set preference parameters at certain empirical values and examine which force will dominate, and to what extent.

What are the empirical values of preference parameters, λ and α ? The existing evidence concerning parameter λ is relatively rich and remarkably consistent: both experimental data (e.g., Kahneman *et al.*, 1990; Tversky and Kahneman, 1991, 1992; Novemsky and Kahneman, 2005) and real data (e.g., Putler, 1992; Hardie *et al.*, 1993) suggest a number close to 2. This is true even for monkeys (Chen *et al.*, 2006). So in the following analysis, we fix λ at 2.25, the value estimated by Tversky and Kahneman (1992).

Table 2.4 Quantitative Analysis

PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$ and $MomEffect = E(R_{t+1} | \theta_t = \theta_H) - E(R_{t+1} | \theta_t = \theta_L)$. WML is the simulated average momentum portfolio return in the multi-stock setting. $Q_t = 1 - H_2(S_t, 1)$ is the turnover, or aggregate selling, in period t . Technology parameter values are fixed at the values in Table 2.1: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. Loss aversion parameter λ is set at 2.25, the value estimated by Tversky and Kahneman (1992). The empirical values of PGR/PLR and momentum are taken from Dhar and Zhu (2006) and Jegadeesh and Titman (1993), respectively. The empirical values of $Corr(R_t, Q_t)$ and $E(R_t - R_f)$ are based on AMEX/NYSE data from 1926-2006.

	$\alpha = 0.37$	$\alpha = 0.52$	$\alpha = 0.88$	Empirical Value
(i) Disposition Effect				
PGR	0.40	0.41	0.41	0.38
PLR	0.18	0.23	0.37	0.17
<i>DispEffect</i>	2.25	1.75	1.10	2.24
(ii) Momentum Effect				
$E(R_{t+1} \theta_t = \theta_H)$	1.1575	1.1431	1.1091	—
$E(R_{t+1} \theta_t = \theta_L)$	1.0822	1.0927	1.1004	—
<i>MomEffect</i>	7.54%	5.04%	0.87%	—
<i>WML</i>	7.20%	4.76%	0.76%	8.60%
(iii) Turnover				
$Corr(R_t, Q_t)$	0.84	0.88	0.91	0.28
(iv) Equity Premium				
$E(R_t - R_f)$	8.14%	7.94%	6.62%	7.84%

But there is not much evidence as to the value of α . As far as we know, only two studies have estimated this parameter, and the results differ markedly in the data sets used. Tversky and Kahneman (1992) estimate $\alpha = 0.88$ by offering subjects isolated gambles in experimental settings. Wu and Gonzalez (1996) use a different experimental data set and estimate $\alpha = 0.52$, but when they apply Camerer and Ho's (1994) data, they find $\alpha = 0.37$. Due to the small sample size in the experiments, none of those studies can estimate α with great precision. So our strategy is to report results for all these three possible values of α in Table 2.4.

Table 2.4 also presents the historical values for the disposition effect, the momentum effect and the correlation between returns and volumes. Unlike Odean (1998), who studies the disposition effect by aggregating across investors, Dhar and Zhu (2006) examine the disposition effect at the level of the individual. They report, in their Table 2.2, that the means of PGR and PLR for all individuals are 0.38 and 0.17, respectively. We adopt these numbers as the empirical values of PGR and PLR. Regarding the momentum effect, we use Jegadeesh and Titman's (1993) estimate, that is, 8.60%, on an annual basis. Using AMEX/NYSE data from 1926-2006 from CRSP, we find that the correlation between returns and volumes, $Corr(R_t, Q_t)$, and the equity premium, $E(R_t - R_f)$, for a typical firm, are 0.28 and 7.84%, respectively.²⁸

²⁸More precisely, we take all stocks in the CRSP database for which at least 11 consecutive years of return and volume data are recorded, compute the correlation between real returns and volume as well as the mean returns in excess of the 30-day T-bill rate for each, and then calculate the medians.

Those historical values help us to evaluate how well our model matches the data. Even though, because we are not confident of the actual value of α among real investors, this evaluation should be interpreted with caution, our quantitative analysis makes a methodological contribution: a general equilibrium model, such as the one provided in the present paper, is the only way to link prospect theory preference to momentum, thereby explaining how much prospect theory preference can contribute to price momentum.

Table 2.4 demonstrates that, for all the three possible values of α , the diminishing sensitivity component of prospect theory dominates the loss aversion component. In particular, when $\alpha = 0.37$, our model matches the historical data well, except for the dimension of the correlation between returns and volumes. To be specific, for $\alpha = 0.37$, our model predicts that $DispEffect = 2.25$, $WML = 7.20\%$ and $E(R_t - R_f) = 8.14\%$, while the historical counterparts for these variables are 2.24, 8.60% and 7.84%, respectively. The model predicts too high a correlation between returns and volumes, i.e., $Corr(R_t, Q_t) = 0.84$, but the empirical value is 0.28.

2.4.4.2 Testable Predictions

One testable prediction emerges from Table 2.4, which suggests that prospect theory simultaneously predicts momentum and a positive correlation between returns and volumes. So we expect the momentum effect to be stronger among those stocks whose returns are *positively* correlated with their own trading volumes.²⁹ This empirical prediction is unique to our mechanism and is

²⁹Note that we don't claim that momentum profits are monotonically increasing in $Corr(R_t, Q_t)$. Actually, Table 4 suggests that the opposite is true.

easy to test. We can rely on this prediction to differentiate our story from other explanations of price momentum, such as the belief-based models proposed by Barberis *et al.* (1998), Daniel *et al.* (1998) or Hong and Stein (1999). Note that our prediction is different from that of Lee and Swaminathan (2000), who show that price momentum is more pronounced among those stocks with higher *levels* of trading volumes, while our predictions relates momentum to the *sensitivity* of returns to volumes.

Besides the above new prediction, our model also makes certain predictions which are consistent with the existing studies. For example, we do not expect prospect theory utility to be equally important for all investors, expecting it to matter more for individual investors than for institutional investors. Indeed, some empirical studies find that mutual fund managers are less prone to the disposition effect than individual investors: the difference between PGR and PLR is 3% for managers, and 5% for retail investors (c.f. Shefrin, 2008). Since our results on momentum are completely driven by prospect theory, one prediction of our model is that a stronger momentum effect will exist among stocks with greater individual investor ownership. Hur *et al.* (2008) test precisely this prediction with a large sample of NYSE/AMEX/NASDAQ stocks between 1981 and 2005 and find strong evidence for this hypothesis. Further evidence comes from Hong *et al.* (2000) and Fama and French (2008), who find that the profitability of momentum strategies declines sharply with market capitalization; since small firms are traded more heavily by individuals, this finding is consistent with our prediction.

Table 2.5 Sensitivity Analysis w.r.t Dividend Growth Rate Volatility $\sigma(\theta_{t+1})$

PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$ and $MomEffect = E(R_{t+1} | \theta_t = \theta_H) - E(R_{t+1} | \theta_t = \theta_L)$. WML is the simulated average momentum portfolio return in the multi-stock setting. $Q_t = 1 - H_2(S_t, 1)$ is the turnover, or aggregate selling, in period t . The risk-free rate is set at $R_f = 1.0386$. The preference parameters are $\alpha = 0.52$ and $\lambda = 2.25$.

	$\theta_L = 0.81586$ $\theta_H = 1.2289$	$\theta_L = 0.7628$ $\theta_H = 1.2821$	$\theta_L = 0.70865$ $\theta_H = 1.3362$
(i) Disposition Effect			
PGR	0.41	0.41	0.41
PLR	0.24	0.23	0.23
<i>DispEffect</i>	1.71	1.75	1.79
(ii) Momentum Effect			
<i>MomEffect</i>	3.83%	5.04%	6.28%
<i>WML</i>	3.62%	4.76%	5.94%
(iii) Turnover			
$Corr(R_t, Q_t)$	0.88	0.88	0.88
(iv) Equity Premium			
$E(R_t - R_f)$	6.13%	7.94%	9.96%

Our model can also relate momentum to the volatility of cash flow. Table 2.5 examines the effect of varying the volatility of the dividend growth rate. For a binary distribution given by equation (1), the dividend growth rate has a mean equal to $E(\theta_{t+1}) = \frac{\theta_H + \theta_L}{2}$, and a volatility equal to $\sigma(\theta_{t+1}) = \frac{\theta_H - \theta_L}{2}$. In Table 2.5, we maintain $E(\theta_{t+1}) - 1$ at 2.24% and change $\sigma(\theta_{t+1})$ from 21% to 26% to 31%.³⁰

³⁰Barberis and Huang (2001) use COMPUSTAT data to estimate the dispersion in firm-level dividend growth volatilities to be 5 percent. So, we

The preference parameters are set at $\alpha = 0.52$ and $\lambda = 2.25$. Table 2.5 suggests that increasing $\sigma(\theta_{t+1})$ generates stronger momentum effects and higher equity premiums. Since a higher $\sigma(\theta_{t+1})$ is also associated with a higher return volatility, the momentum effect is expected to be stronger among stocks both with higher dividend volatility and with higher return volatility. This observation is in fact consistent with Zhang's (2006) finding that momentum profits are higher among firms with higher cash flow volatility or return volatility.

2.5 Conclusion

In this paper, we propose a general equilibrium model to study the implications of prospect theory for individual trading, security prices and trading volume. We show that, in a general equilibrium setting, different components of prospect theory make very different predictions. The diminishing sensitivity component drives a disposition effect, which in turn leads to momentum in the cross-section of stock returns and a positive correlation between returns and volumes. On the other hand, the loss aversion component predicts exactly the opposite, namely a reversed disposition effect and reversal in the cross-section of stock returns, as well as a negative correlation between returns and volume. In a calibrated economy, when prospect theory preference parameters are set at the values estimated by the previous studies, our model can generate price momentum of up to 7% on an annual basis. One testable empirical prediction unique to our model is that the momentum strategy is most profitable, all else equal, among stocks whose returns are positively correlated with their trading volumes.

choose 5% as a step.

APPENDIX

Appendix 2.A.1 Sensitivity Analysis

We have mentioned that generations in our model should be understood as generations of trades, so that one period corresponds to six months to one year. So far in our analysis, we have taken one period to be one year. Table 2.A1 analyzes the effect of changing this assumption, by assuming the decision interval of an investor to be six months. We recalibrate dividend parameters as $\theta_H = 1.19$ and $\theta_L = 0.83$, so that the time-aggregated annual growth rate of dividends has the same mean and volatility as the data. We also reset $R_f - 1$ to be 1.91 percent to maintain a net annual risk-free rate of 3.86 percent. The loss aversion parameter is still set at $\lambda = 2.25$, and the diminishing sensitivity parameter α can take three values: 0.37, 0.52 and 0.88. The variable $WML2$ is the simulated average cumulative annualized momentum portfolio returns:

$$WML2 = \frac{1}{T} \sum_{t=1}^T (R_t^{winner} R_{t+1}^{winner} - R_t^{loser} R_{t+1}^{loser})$$

Comparing Table 2.A1 with Table 2.4, where one period is assumed to be one year, we find that changing the length of the decision interval affects the momentum effect and the equity premium. When the decision interval becomes shorter, a typical investor will experience more losses in one year, and since he is averse to losses, he will demand a higher premium. The higher equilibrium equity premium or, equivalently, the lower price-dividend ratio, means that the disposition effect, i.e., age-2-1 investors' different behavior

facing good news versus bad news, will have a higher impact on the stock return predictability, thereby generating higher returns to the winners-minus-losers portfolio.

As described in Section 2.2, we suppose that investor i uses $R_f^2 W_{1,i}$ as a reference level of wealth when calculating gains and losses. Odean (1998) and Genesove and Mayer (2001) assume that the investor uses the original purchase price as a reference point. That is, if an investor buys a stock at price P^B and sells at price P^S , he calculates gains/losses X as follows: if he holds the stock one period and receives a dividend D , then he perceives $X = P^S + D - P^B$; if he holds the stock two periods and collects dividends D and D' , then he perceives $X = P^S + D + D' - P^B$. Table 2.A2 presents the results for this specification of gains/losses. We still take one period as one year, and the parameter values are fixed at $\theta_H = 1.28$, $\theta_L = 0.76$, $R_f = 1.0386$ and $\lambda = 2.25$. Comparing Table 2.A2 with Table 2.4, we find that this alternative definition of gains/losses has virtually no effect on our results except to deliver a lower equity premium. The reason for the low equity premium is that stock returns don't need to beat the risk-free rate to be counted as gain, which in turn makes the investor more willing to purchase a stock.

Table 2.A1 Results for a Decision Interval of Six Months

The decision interval of the investor is assumed to be six months. Dividend parameters are recalibrated as $\theta_H = 1.1913$ and $\theta_L = 0.8309$, so that the annualized dividend growth rate has a mean of 2.24% and a volatility of 25.97%. The risk-free rate is set as $R_f - 1 = 1.91\%$. Loss aversion parameter λ is set at 2.25. PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$. $WML2$ is the simulated average cumulative annualized momentum portfolio return. $E(R_t R_{t+1} - R_f^2)$ is the annualized equity premium. $Q_t = 1 - H_2(s_t, 1)$ is the turnover, or aggregate selling, in period t .

	$\alpha = 0.37$	$\alpha = 0.52$	$\alpha = 0.88$
(i) Disposition Effect			
PGR	0.40	0.41	0.40
PLR	0.18	0.24	0.37
<i>DispEffect</i>	2.15	1.68	1.07
(ii) Momentum Effect			
<i>WML2</i>	10.33%	6.59%	0.78%
(iii) Turnover			
$Corr(R_t R_{t+1}, \frac{Q_t + Q_{t+1}}{2})$	0.87	0.88	0.89
(iv) Equity Premium			
$E(R_t R_{t+1} - R_f^2)$	11.27%	11.22%	9.69%

When we extend our model to a multi-stock setting and construct the winners-minus-losers portfolio, we have assumed that investors engage in narrow-framing. Is it plausible that people frame individual stocks narrowly? As argued by Barberis and Huang (2007), narrow framing is related to non-consumption utility such as regret: if one of the investor's stocks performs poorly, he may regret the specific decision to buy that stock. So, from a theoretical perspective, gains and losses on individual stocks can affect the investor's decisions. In addition, the extensive empirical evidence on the disposition effect documents that investors, including institutional investors, are reluctant to take losses on the level of individual stocks, suggesting that investors engage in narrow framing in the real market. Of course, a framework that allows the investor to derive utility directly from trading profits on individual stocks, but also, as in traditional models, to derive utility from consumption, namely a framework that allows for both narrow and traditional broad framing at the same time, might fit the data better. Although to construct such a formal model poses significant technical challenges and is beyond the scope of our current analysis, we believe our intuition will carry over, and our main results will survive in this more general setting, so long as the investor's preference can be partially captured by narrow framing and prospect theory, which are the two main drivers of our results.

Table 2.A2 Results for Using Purchase Prices as Reference Points

The investor uses the purchase price as the reference point when calculating capital gains or losses. PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$ and $MomEffect = E(R_{t+1} | \theta_t = \theta_H) - E(R_{t+1} | \theta_t = \theta_L)$. WML is the simulated average momentum portfolio return in the multi-stock setting. $Q_t = 1 - H_2(S_{t,1})$ is the turnover, or aggregate selling, in period t . Technology parameter values are fixed at their values in Table 2.1: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. Loss aversion parameter λ is set at 2.25, the value estimated by Tversky and Kahneman (1992).

	$\alpha = 0.37$	$\alpha = 0.52$	$\alpha = 0.88$
(i) Disposition Effect			
PGR	0.40	0.41	0.49
PLR	0.18	0.24	0.43
<i>DispEffect</i>	2.24	1.74	1.16
(ii) Momentum Effect			
<i>MomEffect</i>	7.39%	4.91%	0.86%
<i>WML</i>	7.07%	4.65%	0.68%
(iii) Turnover			
$Corr(R_t, Q_t)$	0.84	0.88	0.95
(iv) Equity Premium			
$E(R_t - R_f)$	3.86%	3.77%	2.76%

Appendix 2.A.2 Heterogeneity, Aggregation and Price Impacts

In our model, all investors have prospect theory preferences, the preference parameters (α and λ) are the same across investors, and the disposition investors (age-2-1 investors) frame gains/losses in the same way. In reality, investors are likely to be heterogeneous in a number of ways. First, some of them might be better described by traditional, risk-averse expected utility preferences, for example, the standard power utility representation, and these investors might take advantage of prospect theory investors and kill their effects on prices. Second, even prospect theory investors may differ in many dimensions, and this heterogeneity might somehow cause their aggregate behaviors to wash out. So recognizing these heterogeneities raises the question of whether the results of our model still hold in this more realistic world.

A full analysis of this issue poses significant technical hurdles, but there is good reason to believe that a more general model might deliver similar results. On the one hand, as pointed out in the limits to arbitrage literature, there might be limits to the ability and willingness of traditional expected utility maximizers, or arbitrageurs, to offset the pricing effects of prospect theory investors, because by exploiting prospect theory investors, arbitrageurs face fundamental risk as well as noise trader risk, over and above the significant implementation costs they have to bear.³¹ As a result, arbitrageurs will trade

³¹See Barberis and Thaler (2003) Section 2.2, Barberis and Huang (2001) Section IV B, or Barberis and Huang (2008) Section III F for more detailed discussion of this point.

cautiously and only partially absorb the impact on prices of prospect theory investors, thereby allowing our results to persist.

On the other hand, even though prospect theory investors might be heterogeneous in many ways, their disposition related tradings are likely to be systematic and have implications for stock prices. For example, empirical evidence documents that both institutional investors and individual investors exhibit a disposition effect, although the former do so to a smaller extent. This suggests that prospect theory can indeed capture the preferences of both type of investors, albeit differently. Formally, we can model their preferences as prospect theory utility with different parameters (α and λ), or as a combination of consumption utility and prospect theory utility with different weights. This kind of heterogeneity should not wash out in the aggregate, so that prospect theory preferences should have pricing implications. Actually, Coval and Shumway (2005) have provided strong evidence that prospect theory investors indeed move prices.

Another heterogeneity of prospect theory investors relates to the framing of gains/losses. One may argue that different investors might buy into stocks at different prices, so that, in a given period, some investors face gains and others face losses, causing their disposition related tradings to cancel out in aggregate. However, this argument is flawed because it ignores the updating of reference points. When the investor has held a stock many periods, it is more reasonable for him to think of the reference point as some weighted average of the purchase price and other former prices. Once this updating process is taken into account, then in a rising (falling) market, most investors

holding the stock will accumulate gains (losses), regardless of when they bought into the stock or at what price, making their disposition related tradings systematic. This idea can be formalized in a setup with more than three generations. It will, however, exponentially increase the dimension of state vector, making the problem intractable.

REFERENCES

- Admati, Anat, and Paul Pfleiderer. 1988. A Theory of Intraday Patterns: Volume and Price Variability. *Review of Financial Studies* 1(1): 3--40.
- Barberis, Nicholas, and Ming Huang. 2001. Mental Accounting, Loss Aversion, and Individual Stock Returns. *Journal of Finance*, 56(4): 1247-1292.
- Barberis, Nicholas, and Ming Huang. 2007. The Loss Aversion / Narrow Framing Approach to the Equity Premium Puzzle. In *Handbook of Investments: Equity Risk Premium*, ed. Rajnish Mehra, 199-236. Amsterdam: Elsevier Academic Press.
- Barberis, Nicholas, and Ming Huang. 2008. Stocks as Lotteries: The Implications of Probability Weighting for Security Prices. *American Economic Review*, 98(5): 2066--2100.
- Barberis, Nicholas, Ming Huang, and Tano Santos. 2001. Prospect Theory and Asset Prices. *Quarterly Journal of Economics*, 116(1): 1-53.
- Barberis, Nicholas, Andrei Shleifer, and Robert Vishny. 1998. A Model of Investor Sentiment. *Journal of Financial Economics*, 49(3): 307-343.
- Barberis, Nicholas, and Richard Thaler. 2003. A Survey of Behavioral Finance. In *Handbook of the Economics of Finance*, ed. George Constantinides, Milton Harris and René Stulz, 1053-1128. Amsterdam: Elsevier Academic Press.
- Barberis, Nicholas and Wei Xiong. 2008. Realization Utility.
<http://badger.som.yale.edu/faculty/ncb25/rg29b.pdf>.
- Barberis, Nicholas and Wei Xiong. 2009. What Drives the Disposition Effect? An Analysis of a Long-standing Preference-based Explanation. *Journal of Finance*, 64(2): 751-784.

- Benartzi, Shlomo and Richard Thaler. 1995. Myopic Loss Aversion and the Equity Premium Puzzle. *Quarterly Journal of Economics*, 110(1): 73-92.
- Brown, Philip, Nick Chappel, Ray Da Silva Rosa and Terry Walter. 2006. The Reach of the Disposition Effect: Large Sample Evidence Across Investor Classes. *International Review of Finance*, 6(1-2): 43-78.
- Camerer, Colin, and Teck-Hua Ho. 1994. Violations of the Betweenness Axiom and Nonlinearity in Probability. *Journal of Risk and Uncertainty*, 8(2): 167--196.
- Chen, Keith, Venkat Lakshminarayanan, and Laurie Santos. 2006. How Basic Are Behavioral Biases? Evidence from Capuchin Monkey Trading Behavior. *Journal of Political Economy*, 114(3):517-537.
- Coval, Joshua, and Tyler Shumway. 2005. Do Behavioral Biases Affect Prices? *Journal of Finance*, 60(1): 1--34.
- Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam. 1998. Investor Psychology and Security Market Under- and Overreactions. *Journal of Finance*, 53(6): 1839-1885.
- Dhar, Ravi, and Ning Zhu. 2006. Up Close and Personal: Investor Sophistication and the Disposition Effect. *Management Science*, 52(5): 726-740.
- Fama, Eugene, and Kenneth French. 2008. Dissecting Anomalies. *Journal of Finance*, 63(4): 1653-1678.
- Feng, Lei, and Mark Seasholes. 2005. Do Investor Sophistication and Trading Experience Eliminate Behavioral Biases in Finance Markets? *Review of Finance*, 9(3): 305-351.
- Frazzini, Andrea. 2006. The Disposition Effect and Underreaction to News. *Journal of Finance*, 61(4): 2017-2046.

- Garvey, Ryan, and Anthony Murphy. 2004. Are Professional Traders too Slow to Realize Their Losses? *Financial Analysts Journal*, 60(4): 35--43.
- Genesove, David, and Christopher Mayer. 2001. Loss Aversion and Seller Behavior: Evidence from the Housing Market. *Quarterly Journal of Economics*, 116(4): 1233-1260.
- Gomes, Francisco. 2005. Portfolio Choice and Trading Volume with Loss Averse Investors. *Journal of Business*, 78(2): 675-706.
- Griffin, John, Federico Nardari, and René Stulz. 2007. Do Investors Trade More When Stocks Have Performed Well? Evidence from 46 Countries. *Review of Financial Studies*, 20(3): 905-951.
- Grinblatt, Mark, and Bing Han. 2005. Prospect Theory, Mental Accounting, and the Disposition Effect. *Journal of Financial Economics*, 78(2): 311-339.
- Grinblatt, Mark, and Matti Keloharju. 2001. What Makes Investors Trade? *Journal of Finance*, 56(2): 589-616.
- Hardie, Bruce, Eric Johnson, and Peter Fader. 1993. Modeling Loss Aversion and Reference Dependence Effects on Brand Choice. *Marketing Science*, 12(4): 378-394.
- Heath, Chip, Steven Huddart, and Mark Lang. 1999. Psychological Factors and Stock Option Exercise. *Quarterly Journal of Economics*, 114(2): 601-627.
- Hens, Thorsten, and Martin Vlcek. 2006. Does Prospect Theory Explain the Disposition Effect? <http://www.iew.unizh.ch/wp/iewwp262.pdf>.
- Hong, Harrison, Terence Lim, and Jeremy Stein. 2000. Bad News Travels Slowly: Size, Analyst Coverage, and the Profitability of Momentum Strategies. *Journal of Finance*, 55(1): 265-295.
- Hong, Harrison, and Jeremy Stein. 1999. A Unified Theory of Underreaction,

- Momentum Trading, and Overreaction in Asset Markets. *Journal of Finance*, 54(6):2143-2184.
- Hur, Jungshik, Mahesh Pritamani, and Vivek Sharma. 2008. Momentum and the Disposition Effect: The Role of Individual Investors. http://www.fma.org/Texas/Papers/Disp_Investor_Momentum_FMA.pdf.
- Jegadeesh, Narasimhan, and Sheridan Titman. 1993. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance*, 48(1): 65-91.
- Kahneman, Daniel, Jack Knetsch, and Richard Thaler. 1990. Experimental Tests of the Endowment Effect and the Coase Theorem. *Journal of Political Economy*, 98(6): 1325-1348.
- Kahneman, Daniel, and Amos Tversky. 1979. Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, 47(2): 263-291.
- Kaustia, Markku. Forthcoming. Prospect Theory and the Disposition Effect. *Journal of Financial and Quantitative Analysis*.
- Kumar, Alok, and Sonya Lim. 2008. How Do Decision Frames Influence the Stock Investment Decisions of Individual Investors? *Management Science*, 54: 1052-1064.
- Kyle, Albert, Hui Ou-Yang, and Wei Xiong. 2006. Prospect Theory and Liquidation Decisions. *Journal of Economic Theory*, 129(1): 273-285.
- Lee, Charles, and Bhaskaran Swaminathan. 2000. Price Momentum and Trading Volume. *Journal of Finance*, 55(5): 2017-2069.
- Liu, Xiaolei, and Lu Zhang. 2008. Momentum Profits, Factor Pricing, and Macroeconomic Risk. *Review of Financial Studies*, 21(6):2417-2448.
- Locke, Peter, and Steven Mann. 2005. Professional Trade Discipline and Trade Disposition. *Journal of Financial Economics*, 76(2): 401--444.

- Novemsky, Nathan, and Daniel Kahneman. 2005. The Boundaries of Loss Aversion. *Journal of Marketing Research*, 42(2): 119-128.
- Odean, Terrance. 1998. Are Investors Reluctant to Realize Their Losses? *Journal of Finance*, 53(5): 1775-1798.
- Oehler, Andreas, Klaus Heilmann, Volker Lager, and Michael Oberlander. 2002. Michael, Dying Out Or Dying Hard? Disposition Investors In Stock Markets. <http://ssrn.com/abstract=314139>.
- Putler, Daniel. 1992. Incorporating Reference Price Effects into a Theory of Consumer Choice. *Marketing Science*, 11(3): 287-309.
- Radner, Roy. 1972. Existence of Equilibrium of Plans, Prices, and Price Expectations in a Sequence of Markets. *Econometrica*, 40(2): 289-303.
- Scherbina, Anna, and Li Jin. 2006. Inheriting Losers. <http://www.people.hbs.edu/ascherbina/dispositionJune22006.pdf>.
- Shapira, Zur, and Itzhak Venezia. 2001. Patterns of Behavior of Professionally Managed and Independent Investors. *Journal of Banking and Finance*, 25(8): 1573-1587.
- Shefrin, Hersh. 2008. *A Behavioral Approach to Asset Pricing*. Amsterdam: Elsevier Academic Press.
- Shefrin, Hersh, and Meir Statman. 1985. The Disposition to Sell Winners Too Early and Ride Losers Too Long: Theory and Evidence. *Journal of Finance*, 40(3): 777-790.
- Shumway, Tyler, and Guojun Wu. 2007. Does Disposition Drive Momentum. <http://www.bauer.uh.edu/wu/Papers/Disposition.pdf>.
- Statman, Meir, Steven Thorley, and Keith Vorkink. 2006. Investor Overconfidence and Trading Volume. *Review of Financial Studies*, 19(4):, 1531-1565.

- Thaler, Richard. 1980. Toward a Positive Theory of Consumer Choice. *Journal of Economic Behavior and Organization*, 1(1): 39-60.
- Thaler, Richard. 1985. Mental Accounting and Consumer Choice. *Marketing Science*, 4(3): 199-214.
- Tversky, Amos, and Daniel Kahneman. 1991. Loss Aversion and Riskless Choice: A Reference Dependent Model. *Quarterly Journal of Economics*, 106(4): 1039-1061.
- Tversky, Amos, and Daniel Kahneman. 1992. Advances in Prospect Theory: Cumulative Representation of Uncertainty. *Journal of Risk and Uncertainty*, 5(4): 297-323.
- Weber, Martin, and Colin Camerer. 1998. The Disposition Effect in Securities Trading: An Experimental Analysis. *Journal of Economic Behavior and Organization*, 33(2): 167-184.
- Wermers, Russell. 2003. Is Money Really `Smart'? New Evidence on the Relation Between Mutual Fund Flows, Manager Behavior, and Performance Persistence. <http://ssrn.com/abstract=414420>.
- Wu, George, and Richard Gonzalez. 1996. Curvature of the Probability Weighting Function. *Management Science*, 42(12): 1676--1690.
- Zhang, Frank. 2006. Information Uncertainty and Stock Returns. *Journal of Finance*, 61(1): 105-137.

CHAPTER 3

DIVIDEND VOLATILITY and ASSET PRICING³²

3.1 Introduction

How does aggregate dividend volatility affect asset prices?³³ Until now the literature has largely disregarded this question. To the best of our knowledge, the only exception is Longstaff and Piazzesi (2004), who demonstrate that volatile and procyclical dividends can raise equity premiums in a representative agent model with power utility. However, their model explains less than half as large as historical equity premiums, and they don't explore whether dividend volatility can help explain other puzzling facts in the aggregate stock market, such as return predictability and time-varying Sharpe ratios. More importantly, their consumption-based model will inevitably predict a high correlation between consumption and stock returns, contradicting our observation. In this paper, we turn to a narrow-framing approach to comprehensively study the pricing implications of dividend volatility, and find that our model can explain key asset markets phenomena.

Narrow-framing means that, when people evaluate risk, they often appear to pay attention to narrowly defined gains and losses. This behavior is uncovered by experimental work on decision-making under risk (e.g., Kahneman and

³²This chapter is based on a joint paper with Yan Li.

³³Throughout the paper, the term dividend volatility refers to the standard deviation of the *growth rate* of (not the level of) the *aggregate* dividends paid to all stocks. See equation (4) for a technical definition.

Lovall, 1993; Kahneman, 2003). In the context of financial investment, narrow-framing states that investors tend to separate their financial wealth from their overall wealth, and are inclined to get utility directly from fluctuations in the value of their overall portfolio of stocks (Benartzi and Thaler, 1995; Barberis and Huang, 2001; Barberis, Huang, and Santos, 2001, henceforth BHS; Barberis and Huang, 2007).³⁴ Under this assumption, investors may perceive aggregate dividend volatility, which drives fluctuations in the value of their financial wealth, as a more appropriate metric to represent risk than consumption volatility, a commonly used measure in the literature. This immediately implies that dividend volatility has significant implications for asset prices.

In this paper, we first provide strong empirical evidence that (i) dividend volatility exhibits strong persistence, usually called volatility clustering, indicating the tendency of a big (small) change today to be followed by a big (small) change tomorrow, (ii) dividend volatility has declined dramatically in the postwar period.³⁵ The aggregate dividend time series we use is backed out from CRSP stock return data.³⁶ This imputed dividend series has accounted for stock repurchases as an increasingly significant component of dividends

³⁴In the literature, narrow-framing is sometimes applied to individual stocks that investors own (e.g., Barberis and Huang, 2001). For a deep discussion on narrow-framing, see Barberis and Huang (2007).

³⁵Lettau, Ludvigson and Wachter (2008) also mention that the volatility of dividend growth has declined since 1990s. But their model assumes that this decline affects stock prices through consumption.

³⁶This constructed dividend index is identical to Campbell (2000). The detailed data construction is given in the appendix.

since 1980. One may argue that the declining trend in dividend volatility is partly due to corporate managers' intention to smooth dividend. Whatever the reason is, however, an investor in our theoretical model takes the dividend process as exogenously given when making her investment decisions, which is a standard assumption in the asset pricing literature.

We further propose a theoretical model in which dividend volatility is persistent and investors exhibit loss aversion: they dislike fluctuations in their financial wealth; and the more persistent the dividend volatility, the more they dislike stocks. Loss aversion is a central feature of the prospect theory of Kahneman and Tversky (1979), which is based on a variety of experimental evidence and has been extensively used in behavioral finance literature (e.g., Benartzi and Thaler, 1995; BHS, 2001).

Our model is able to account for many of the stylized facts of asset prices, including the high mean and excess volatility of stock prices, predictability of stock returns, time-varying Sharpe ratios, a low and stable risk-free rate, and the low correlation between consumption and stock returns. Our model shows, moreover, the substantial decline in dividend volatility since the 1950s, signals a much more stable investment environment, which loss averse investors prefer; they therefore require a much lower return on holding stocks, resulting in lower equity premiums. This is consistent with Blanchard (1993), Fama and French (2002), and Buranavityawut, Freeman and Freeman (2006), who find that ex-ante equity premiums have declined in the past fifty years.

Dividend volatility plays an essential role in explaining the intuitions of our model. As the state variable, it completely determines equilibrium price-

dividend ratios and helps explain the high mean and excess volatility of stock returns. In equilibrium, a rise (drop) in dividend volatility lowers (raises) asset prices, and hence price-dividend ratios fluctuate with the dividend volatility process, generating excess volatility in market returns. The high volatility of returns, in turn, means that stocks often perform poorly, causing loss averse investors considerable discomfort and leading to low stock prices or high risk premiums. Furthermore, dividend volatility tends to be higher in market troughs than in booms, which leads to the countercyclical expected excess returns observed in financial markets.

The persistence of dividend volatility leads to the persistence of the price-dividend ratio, producing predictability in stock returns, where the forecasting power increases with the forecast horizon. The conditional mean and conditional standard deviation of expected returns are driven differently by dividend volatility, hence the Sharpe ratio as a measure of the price of risk changes over time. Moreover, the model-generated stock returns correlate only weakly with consumption, because stock returns are ultimately driven by dividend news, which has a low correlation with consumption news.

Many studies have been devoted to explaining these puzzling facts in the literature. Our work is closely related to two prominent approaches,³⁷ but also differs in a variety of ways. The first approach, including Campbell and

³⁷Besides the two approaches mentioned here, another line of research relies on modifying the market and asset structure (e.g., Constantinides and Duffie, 1996; Heaton and Lucas, 1996).

Cochrane (1999) and BHS (2001), relies on stochastic changing risk aversion, whereas the second, including Bansal and Yaron (2004, henceforth BY), relies on the changing economic environment.

With respect to the first approach, we share with BHS (2001) the use of loss aversion to describe investors' preferences. However, we depart from them in two ways: we use loss aversion as the only psychological assumption, and our result isn't driven by the changing risk aversion of investors. BHS's result depends crucially on another psychological assumption, usually labelled the house money effect, which refers to the experimental finding that people are more (less) willing to bear risks when they have had prior gains (losses). The house effect together with loss aversion generates their model's results.

In terms of the mechanism, our model is similar to BY (2004) in that we all require a persistent component in the underlying processes. However, our model specification is less stringent than theirs. In BY's model, it's critical to model the growth rates of both consumption and dividends as containing a long-run predictable component, as well as containing persistent volatility to stand for fluctuating economic uncertainty. In conjunction with Epstein and Zin's (1989) preferences, they succeed in explaining the financial market phenomena. However, as BY have pointed out, since it's econometrically difficult to distinguish an i.i.d. process from a process containing a small persistent component, it's rather difficult to justify the forecastable persistent component in the consumption and dividend growth rates. In our model, we need the persistent component only in the volatility of the dividend growth rate, which is supported by strong econometric evidence; we don't rely on the persistent component in the growth rates *per se*, which lacks empirical

evidence. The consumption growth rate is still maintained to be a white noise process in our model.

The rest of the paper is organized as follows. Section 3.2 provides extensive econometric evidence to show that (i) dividend volatility is persistent over time and (ii) it changes with the business cycle and experiences significant declines in the postwar period. Section 3.3 presents the model and characterizes the equilibrium asset prices. Section 3.4 calibrates the model and solves the price-dividend ratios, then analyzes model simulation results. Section 3.5 concludes the paper.

3.2. Key Features of Historical Dividend Volatility

3.2.1 Dividend Volatility Clustering

In this subsection, we provide evidence that dividend volatility displays the property of clustering, which, as we will see more clearly later, plays an important role in explaining the high mean, excess volatility, as well as the predictability of stock returns. We perform a variety of standard econometrics tests: first identify whether volatility clustering in dividend in fact exists and, if so, run a unit-root test to check how strong this persistence is.

Table 3.1 Dividend Volatility Estimates

Panel A reports the test statistics for Box-Pierce-Ljung test and ARCH test for lag=4, 8 and 12 on quarterly dividend growth rate from 1926.Q3 to 2006.Q3.

Panel B models the dividend growth rate, $g_{D,t+1}$, as AR(1)-EAGRCH(1,1), $g_{D,t+1} = \beta_0 + \beta_1 g_{D,t} + \sigma_t Z_{t+1}$, $\log \sigma_t^2 = \kappa + \mathbf{G}_1 \log \sigma_{t-1}^2 + \mathbf{A}_1 [|Z_t| - E|Z_t|] + \mathbf{L}_1 Z_t$, where σ_t^2 is conditional variance of $g_{D,t+1}$, and $Z_{t+1} \sim i.i.d. \mathcal{N}(0,1)$. Panel C reports an augmented Dicky-Fuller test on the log of the conditional volatility series estimated by an AR(1)-EAGRCH(1,1). Panel D models the dividend growth rate as a regime-switching process: $g_{D,t+1} = \mu_{s_t} + \sigma_{s_t} v_{t+1}$, $v_{t+1} \sim \mathcal{N}(0,1)$, where $\mu_{s_t} \in \{\mu_1, \mu_2\}$ and $\sigma_{s_t} \in \{\sigma_1, \sigma_2\}$ depend on the underlying state s_t , which follows a Markov chain characterized by transitional probabilities \mathbf{p}_{11} and \mathbf{p}_{22} . In Panels B and D, the standard errors of the estimated parameters are reported in parentheses. ** and * mean that the estimates are significantly different from zero at 1% and 5% levels, respectively.

Panel A: Dividend Volatility Clustering Tests			
Lag	4	8	12
Box-Pierce-Ljung Test	32.57**	93.29**	101.37**
ARCH Test	24.96**	66.72**	73.58**

Panel B: AR(1)-EAGRCH(1,1) Estimation						
Parameters	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\kappa}$	$\hat{\mathbf{G}}_1$	$\hat{\mathbf{A}}_1$	$\hat{\mathbf{L}}_1$
Values	0.002* (0.001)	0.451** (0.0618)	-0.224* (0.109)	0.968** (0.014)	0.423** (0.077)	0.037 (0.034)

Panel C: Augmented Dicky-Fuller Test			
Test Statistic	Critical Values		
	1%	5%	10%
-11.8	-20.3	-14.0	-11.2

Panel D: Regime-switching Estimation						
Parameters	$\hat{\mu}_1$	$\hat{\sigma}_1$	$\hat{\mu}_2$	$\hat{\sigma}_2$	$\hat{\mathbf{p}}_{11}$	$\hat{\mathbf{p}}_{22}$
Values	0.007** (0.001)	0.018** (0.001)	-0.013 (0.013)	0.078** (0.007)	0.969** (0.007)	0.797** (0.046)

Volatility clustering, which characterizes the persistence in volatility, has been documented as a standard feature of many financial series. For instance, Bollerslev, Engle and Wooldridge (1988) show that conditional variance of market return fluctuates across time and is very persistent. For high-frequency return data, the ARCH literature finds a very high coefficient in the correlation of conditional standard deviations of returns. In our model, we consider volatility clustering in the dividend growth rate and examine its impact on equilibrium asset prices. Even though our data are at a low-frequency, the estimated coefficient is very similar to those found in high-frequency data. Before running the ARCH type tests, we first run two diagnostic tests to see if there is volatility clustering in the dividend growth rate, which is constructed from the value weighted NYSE/AMEX return data from CRSP. More specifically, we use two standard tests in the econometric literature, the Box-Pierce-Ljung test and ARCH test, to check whether there are strong correlations in the second moment of the dividend growth rate. Both tests have as the null hypothesis that there's no volatility clustering in the dividend growth rate, and under the null, both tests asymptotically follow a Chi square distribution. Panel A of Table 3.1 presents the test results. The statistics from both tests significantly reject the null hypothesis, indicating strong persistence in the volatility of the dividend growth rate.

The preliminary tests make us comfortable using the exponential GARCH (EGARCH) model to identify the persistent component in dividend volatility. We use EAGRCH for two reasons: first, it matches best with our theoretical dividend volatility specification in section 3.3; second, it can capture the

asymmetric behaviors in volatility, i.e., larger (smaller) volatility is associated with negative (positive) news. Specifically, we consider the following regression:

$$\begin{aligned} g_{D,t+1} &= \beta_0 + \beta_1 g_{D,t} + \sigma_t Z_{t+1}, \\ \log(\sigma_t^2) &= \kappa + \mathbf{G}_1 \log(\sigma_{t-1}^2) + \mathbf{A}_1 [|Z_t| - E|Z_t|] + \mathbf{L}_1 Z_t, \\ Z_{t+1} &\sim i.i.d. \mathbf{N}(0,1), \end{aligned} \quad (1)$$

where $g_{D,t+1}$ is the dividend growth rate, σ_t^2 is the conditional variance of $g_{D,t+1}$, and $\beta_0, \beta_1, \kappa, \mathbf{G}_1, \mathbf{A}_1$ and \mathbf{L}_1 are coefficients.³⁸ Panel B of Table 3.1 reports the estimation result. In addition to this EGARCH specification, we also try the specifications in Bansal, Khatchatrian and Yaron (2005) and get similar results not reported here.

The coefficient for dividend volatility is $\hat{\mathbf{G}}_1 = 0.968$, indicating that persistent dividend volatility indeed exists, which is consistent with the standard findings in the ARCH literature. However, the coefficient that measures the persistence in the dividend growth rate *per se* is much smaller ($\hat{\beta}_1 = 0.451$). In the long-run risk literature (e.g., BY, 2004; Bansal, Kiku and Yaron, 2007), it is crucial to have the persistence in both the mean and the volatility of the dividend growth rate to explain the high equity premium, in other words, both $\hat{\mathbf{G}}_1$ and $\hat{\beta}_1$ are assumed to be close to one. In contrast, our model requires persistence only in the volatility, but not in the mean of the dividend growth rate process. The

³⁸In what follows, we report results based on this AR(1)-EGARCH(1,1) specification. We also tried AR(2)-EGARCH(1,1) and other specifications, and the main results remain unchanged.

current estimation result shows that the econometric evidence is weak for the persistence in the dividend growth rate, but that the persistence in dividend volatility is strong, providing strong econometric evidence for our model.

To further confirm that the persistence of dividend volatility is indeed very high, we resort to the augmented Dicky-Fuller unit root test by running the following regression:³⁹

$$\log(\hat{\sigma}_t) = \alpha_0 + \alpha_1 \log(\hat{\sigma}_{t-1}) + \alpha_2 \Delta \log(\hat{\sigma}_{t-1}) + e_t$$

where $\hat{\sigma}_t$ is conditional dividend volatility obtained from the EGARCH estimation (equation [1]), and α_0, α_1 and α_2 are coefficients. Panel C of Table 3.1 reports the test statistics together with the critical values at 1%, 5%, and 10% levels. We can hardly reject the null hypothesis of $\alpha_1 = 1$ at the 10% critical level, which implies that dividend volatility is indeed very persistent.⁴⁰ For comparison, we also run the unit root test in the dividend growth rate, and the unreported result strongly rejects the unit root hypothesis at any critical level, which is not surprising given that $\hat{\beta}_1$ is only 0.451 in Panel B of Table 3.1.

³⁹An IGARCH (integrated GARCH) model will be able to nest the EGARCH estimation and the unit root estimation. However, we don't use IGARCH for two reasons: first, IGARCH is not stationary because it assumes a unit root in the volatility process; second, EGARCH fits more with our theoretical dividend volatility specification. We dispense with long memory GARCH models for similar reasons.

⁴⁰The persistence of dividend volatility is going to generate important model results. Therefore, α has to be sufficiently high although it need not be close to 1.

Given the strong econometric evidence, we believe that dividend volatility clustering is an important feature of the actual dividend data. Our theoretical model incorporates this feature when we specify the dividend growth rate process.⁴¹

3.2.2 Time-Varying Dividend Volatility

In this subsection, we examine the evolution of dividend volatility by asking two questions. How does dividend volatility vary with the business cycle? Is there any remarkable change in dividend volatility over the years? Since dividend volatility is the state variable in our model, the answer to the first question will enable us to analyze the procyclical stock prices through the model. The answer to the second question can relate our measure of macroeconomic risk to the measures in other papers, and provide empirical support for our model to explain the dynamics of equity premiums.

To see how dividend volatility varies with the business cycle, Figure 3.1 plots dividend volatility, the real GDP growth rate, and the recession periods identified using NBER's business cycle chronology. In this figure, dividend volatility is the conditional standard deviation estimated from the EGARCH model (equation [1]); the real GDP growth rates are obtained from the website of the Bureau of Economic Analysis and start from the second quarter of 1947; and the shaded areas correspond to the economic recession periods according to NBER's business cycle chronology.

⁴¹That is, we require a high ϕ in equation (5).

We see that dividend volatility changes over time, with the highest values appearing in the 1930's. Comparing dividend volatility with GDP growth rates, we see roughly a negative relationship: high dividend volatility usually coincides with lower GDP growth rates. This pattern makes sense, because it's usually the case that more uncertainty is present when the economy is in a trough. Further comparing it with NBER identified recessions, we find that dividend volatility tends to be very high during most recessions. The evidence suggests that dividend volatility evolves in a counter-cyclical way, which can potentially generate procyclical stock prices as well as counter-cyclical equity premiums and Sharpe ratios. Although this direction is promising, this evidence is weak. We thus take a conservative view in next section, assuming that the dividend volatility process is uncorrelated with the consumption growth process.⁴²

Observing the data through time, Figure 3.1 also shows that dividend volatility was relatively high before 1952 and became much smoother thereafter, except for a spike around 1989. Therefore, dividend volatility seems to have undergone a significant decline in the postwar years, which suggests that investors' perceived⁴³ financial risk, as an inseparable part of macroeconomic risk, has experienced a significant decline since the 1950s.

⁴²That is we assume $Cov(\eta_t, u_t) = 0$ in equation (6).

⁴³Here, perceived is used to emphasize the notion that investors treat the dividend process as exogenous, although firms tend to smooth dividends in reality.

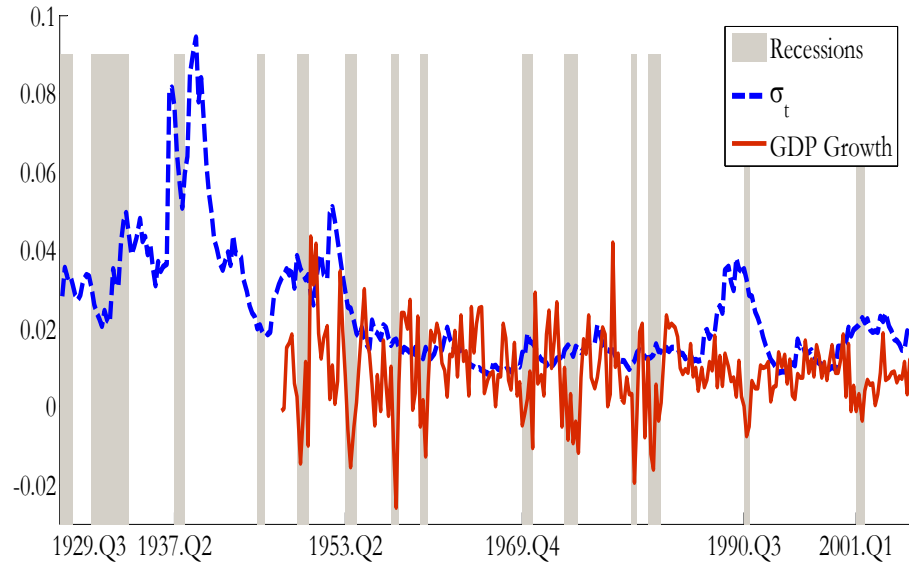


Figure 3.1 Dividend Volatility, GDP Growth and Recessions

Figure 3.1 plots quarterly consumption growth rates for period 1947.Q1-2006.Q3, and conditional dividend volatility for period 1926.Q3-2006.Q3. The dividend volatility σ_t is estimated from an AR(1)-EGARCH(1,1) regression. The shaded bars indicate the recessions according to NBER's website data.

To characterize the decline in dividend volatility more formally, we follow Hamilton (1989) to estimate a regime-switching model. The basic idea is to model the dividend growth rate as deriving from one of two regimes, a regime with a high dividend volatility or one with a low dividend volatility. The parameter values in each regime, together with the transitional probability can be obtained through maximum likelihood estimation. These parameter estimates can then be used to infer which regime the process was in at any historical date. Specifically, the dividend growth rate, $g_{D,t+1}$, is generated

according to:

$$g_{D,t+1} = \mu_{s_t} + \sigma_{s_t} v_{t+1}, \quad v_{t+1} \sim \mathbf{N}(0,1),$$

where $\mu_{s_t} \in \{\mu_1, \mu_2\}$ is the mean, and $\sigma_{s_t} \in \{\sigma_1, \sigma_2\}$ is the volatility in state s_t . Thus, when $s_t = 1$, the observed dividend growth rate, $g_{D,t+1}$, is presumed to have been drawn from a $\mathbf{N}(\mu_1, \sigma_1)$ distribution, whereas when $s_t = 2$, $g_{D,t+1}$, is drawn from another distribution $\mathbf{N}(\mu_2, \sigma_2)$. The state evolves according to a Markov process, and we denote the transitional probability of the Markov chains

$$\begin{aligned} \Pr(s_t = 1 | s_{t-1} = 1) &= \mathbf{p}_{11}, \\ \Pr(s_t = 2 | s_{t-1} = 2) &= \mathbf{p}_{22}. \end{aligned}$$

The parameter values and their standard deviations are reported in Panel D of Table 3.1. The estimated two regimes are characterized as follows: the high-mean, low-volatility regime has an average growth rate of 0.65% per quarter, with a low standard deviation of 0.018; the low-mean, high-volatility regime has an average growth rate of -1.3% per quarter, with a very high standard deviation 0.078. In addition, the high-mean, low-volatility regime seems more persistent, because its transitional probability is higher, $\hat{\mathbf{p}}_{11} = 0.969$.

Figure 3.2 plots the smoothed posterior probability of the dividend growth rate being in a low-mean, high-volatility state. The probability is very high in prewar data, but exhibits sharp declines after the 1950s. In much of the postwar period, the posterior probability of being in a high-mean, low-volatility regime is

close to one.

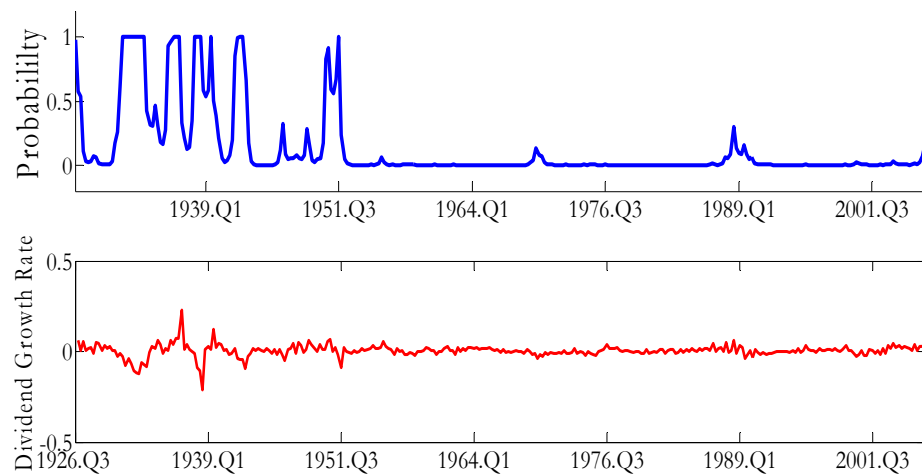


Figure 3.2 Smoothed Probability of a High Volatility Regime and Historical Dividend Growth Rate

In Figure 3.2, the dividend growth rate is assumed to be generated from a regime switching model. The estimation results in Panel D of Table 1 suggests that one regime features a positive mean and a low volatility, while the other one has a negative mean and a high volatility. The top panel plots the posterior probability of dividend growth being in the high volatility regime given the observed data process. The bottom panel plots the dividend growth rate for period 1926.Q3-2006.Q3.

The reported evidence clearly shows that dividend volatility has been declining since 1950s. This is broadly consistent with the findings in Kim, Morley and Nelson (2004), who document a similar pattern in stock returns. In Section 3.4.3.5, we incorporate this finding into our theoretical framework by doing comparative statics with respect to the exogenous parameters governing the dividend process, and find that the declining dividend volatility helps to explain the decreasing equity premiums after WWII.

3.3 The Model

3.3.1 Setup

Consider an economy populated with a continuum of identical, infinitely lived, narrow-framing and loss averse agents. Two assets are available to trade: a risk-free asset in zero net supply, paying a gross interest rate $R_{f,t}$, and one unit of risky asset, paying a gross return R_{t+1} , between time t and $t+1$.

The loss averse investor chooses consumption C_t and risky asset holdings S_t to maximize the utility function

$$E \left[\sum_{t=0}^{\infty} \left(\rho^t \frac{C_t^{1-\gamma}}{1-\gamma} + b_0 \bar{C}_t^{-\gamma} \rho^{t+1} v(X_{t+1}) \right) \right], \quad (2)$$

subject to the standard budget constraint, and

$$X_{t+1} = S_t (R_{t+1} - R_{f,t}),$$

$$v(X_{t+1}) = \begin{cases} X_{t+1}, & \text{if } X_{t+1} \geq 0, \\ \lambda X_{t+1}, & \text{if } X_{t+1} < 0. \end{cases}$$

The first term in the objective function is the standard utility over consumption, where $\rho \in (0,1)$ is the time discount factor; $\gamma > 0$ measures the curvature of the investor's utility over consumption;⁴⁴ and \bar{C}_t is the aggregate per capita consumption at time t , which is exogenous to the investor. The exogenous scalar \bar{C}_t is introduced to ensure that consumption utility and prospect utility

⁴⁴For $\gamma = 1$, we replace $C_t^{1-\gamma}/(1-\gamma)$ with $\log(C_t)$.

are of the same order as aggregate wealth increases over time.⁴⁵

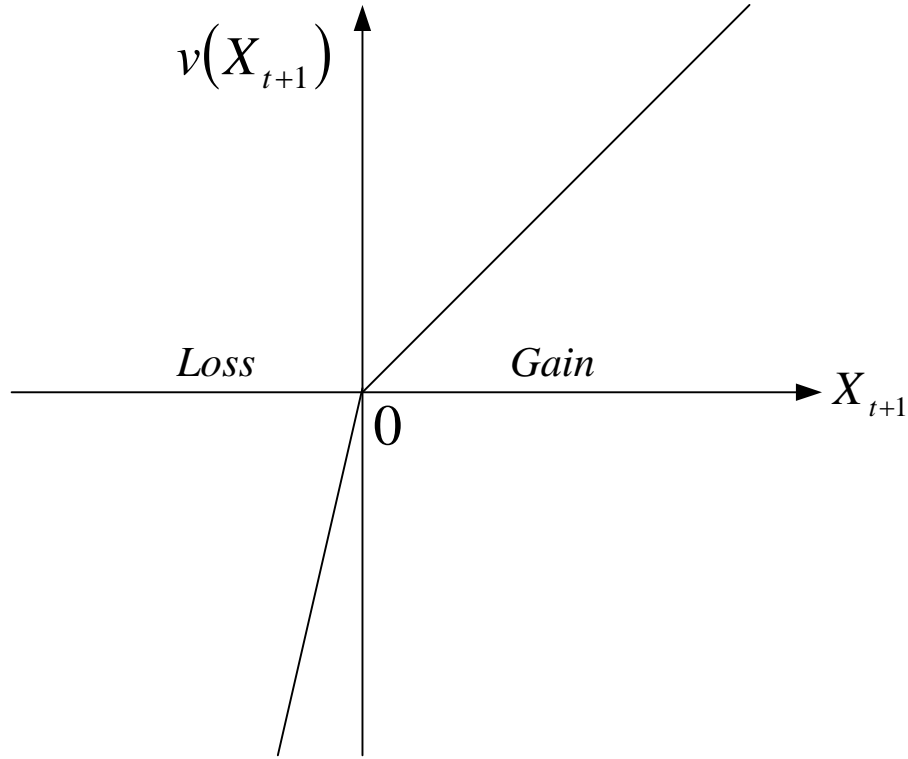


Figure 3.3 Gain and Loss Function

Figure 3.3 plots the gain and loss function $v(X_{t+1}) = \begin{cases} X_{t+1}, & \text{if } X_{t+1} > 0 \\ \lambda X_{t+1}, & \text{if } X_{t+1} \leq 0 \end{cases}$ for $\lambda > 1$.

The second term deserves more attention, as it captures the direct utility the investor derives from fluctuations in the value of her financial wealth.

Depending on the return of the risky asset, her total portfolio excess return X_{t+1}

⁴⁵Another tractable preference specification that incorporates narrow-framing but doesn't rely on a scaling to ensure stationarity can be found in Barberis and Huang (2007).

can be either positive or negative, a positive one indicating a financial gain, and a negative one a financial loss. The function $v(X_{t+1})$ describes how she feels about her investment performance. Since she is loss averse, the pain she receives from financial losses outweighs the happiness from financial gains. Therefore, $v(X_{t+1})$ takes different functional form with respect to the values of X_{t+1} : when X_{t+1} is positive showing that she makes money, $v(X_{t+1})$ is linear in X_{t+1} with slope one; in contrast, when X_{t+1} is negative meaning that she loses money, $v(X_{t+1})$ amplifies her utility loss by a magnitude of λ , with λ being greater than one. Figure 3.3 plots the function $v(X_{t+1})$.

The dynamics of the economy crucially depends on the value of b_0 , which tells how much the second utility counts in her total utility. If $b_0 = 0$, loss aversion doesn't play a role in the overall utility, and the model is reduced to a traditional asset pricing setting studied by Hansen and Singleton (1983). In this case, higher dividend volatility leads to a higher dividend growth rate, resulting in a higher price-dividend ratio and a lower equity premium. However, as the value of b_0 increases, the investor suffers more utility loss from her financial loss and demands a higher risk premium in holding stocks. As will be clearer later, the balance of these two utility forces generates the pattern actually observed in financial markets.

Both consumption and dividend growth follow lognormal processes,

$$g_{C,t+1} = \log(\bar{C}_{t+1} / \bar{C}_t) = g_C + \sigma_C \eta_{t+1}, \quad (3)$$

$$g_{D,t+1} = \log(D_{t+1} / D_t) = g_D + \sigma_D \varepsilon_{t+1}, \quad (4)$$

$$\log(\sigma_{t+1}) - \log(\bar{\sigma}) = \phi[\log(\sigma_t) - \log(\bar{\sigma})] + \sigma_u u_{t+1} \quad (5)$$

Here $g_{C,t+1}$ is the growth rate of aggregate consumption \bar{C}_t . g_C and σ_C are the mean and standard deviation of consumption growth. D_t is dividend: its growth rate is denoted as $g_{D,t+1}$, with mean g_D and standard deviation σ_t . We draw special attention to equation (5), which characterizes the evolution of dividend volatility. To ensure the positiveness of σ_t , we model $\log(\sigma_{t+1})$ instead of σ_t as an $AR(1)$ process. In this sense, the dividend volatility equation (5) is very similar to an EGARCH specification (equation [1]). σ_u captures the magnitude of the innovation to the conditional volatility σ_t : a big σ_u will increase dividend volatility. A particular interesting parameter is the coefficient ϕ , which controls the strength of dependence on past volatilities. A larger ϕ implies that the impact of a shock to dividend volatility is very persistent. As has been shown in Section 3.2, this persistence parameter ϕ is very high in actual dividend data.

The innovations to the consumption growth η_t , the dividend growth ε_t , and the dividend volatility u_t are jointly normally distributed as

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \\ u_t \end{pmatrix} \sim \text{i.i.d. } \mathbf{N} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \omega & 0 \\ \omega & 1 & \theta \\ 0 & \theta & 1 \end{pmatrix} \right], \quad (6)$$

where ω is the correlation between consumption shocks and dividend shocks. Note that when allowing for persistence in dividend volatility, the unconditional correlation between $g_{C,t+1}$ and $g_{D,t+1}$ is $\omega e^{-0.5\sigma_u^2/(1-\phi^2)} < \omega$. As discussed in Section 3.2.2, we assume consumption growth shocks are independent of

dividend volatility shocks, i.e., $Cov(\eta_t, u_t) = 0$, although data suggests a weak negative correlation between η_t and u_t , which has important implications for the time-variation pattern of the equity premiums. We allow for the interaction between shocks to the dividend growth rate ε_t and shocks to the dividend volatility u_t , and the interaction of these two shocks are denoted by θ . As will be shown later, θ also plays a role in generating certain model results.

3.3.2 Equilibrium Prices

This subsection derives the equilibrium asset prices. We first construct a one-factor Markov equilibrium, in which the risk-free rate is a constant and the state variable σ_t (dividend volatility) determines the distribution of future stock returns. Assume that the price-dividend ratio is a function of σ_t :

$$f_t \equiv P_t / D_t = f(\sigma_t).$$

We are going to verify that there is indeed an equilibrium satisfying this assumption.

Given the one-factor assumption, the stock returns R_{t+1} can be determined as

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{1 + P_{t+1}/D_{t+1}}{P_t/D_t} \frac{D_{t+1}}{D_t} = \frac{1 + f(\sigma_{t+1})}{f(\sigma_t)} e^{g_D + \sigma_t \varepsilon_{t+1}}. \quad (7)$$

Intuitively, the change in stock returns can be attributed to either the news about dividend growth ε_{t+1} , or the financial market uncertainty σ_t , or changes in the price-dividend ratio f . Since the dividend process is exogenously given, the key to solving R_{t+1} is to solve the price-dividend ratio f .

In equilibrium, the Euler equations fully capture the dynamics of the economy⁴⁶

$$1 = \rho R_f E_t \left[\left(\bar{C}_{t+1} / \bar{C}_t \right)^{-\gamma} \right] \quad (8)$$

$$1 = \rho E_t \left[R_{t+1} \left(\bar{C}_{t+1} / \bar{C}_t \right)^{-\gamma} \right] + b_0 \rho E_t [\hat{v}(R_{t+1})] \quad (9)$$

where

$$\hat{v}(R_{t+1}) = \begin{cases} R_{t+1} - R_{f,t}, & \text{if } R_{t+1} \geq R_{f,t}, \\ \lambda(R_{t+1} - R_{f,t}), & \text{if } R_{t+1} < R_{f,t}. \end{cases}$$

Equation (8) and the i.i.d. assumption on the consumption growth together imply a constant risk free rate,

$$R_f = \rho^{-1} e^{g_C - \gamma^2 \sigma_C^2 / 2}. \quad (10)$$

After substituting in the respective consumption and dividend processes, equation (9) boils down to

$$1 = \rho E_t \left[\frac{1 + f(\sigma_{t+1})}{f(\sigma_t)} e^{g_D + \sigma_t \varepsilon_{t+1}} e^{-\gamma(g_C + \sigma_C \eta_{t+1})} \right] + b_0 \rho E_t \left[\hat{v} \left(\frac{1 + f(\sigma_{t+1})}{f(\sigma_t)} e^{g_D + \sigma_t \varepsilon_{t+1}} \right) \right], \forall \sigma_t. \quad (11)$$

In equilibrium, the function f must evolve according to equation (11), which

⁴⁶The Euler equations are both necessary and sufficient to characterize the equilibrium. Refer to BHS (2001) for a proof.

also verifies the conjectured one-factor Markov equilibrium price function.

3.3.3 Methodology of Numerical Computation

We solve f numerically on a grid search of the state variable σ_t . We start out by guessing a solution to (11), $f^{(0)}$ say. According to (5), the distribution of σ_{t+1} is completely determined by σ_t and u_{t+1} . Then we get a new candidate solution $f^{(1)}$ by the following recursion

$$1 = \rho e^{g_D - \gamma g_C + \frac{1}{2} \gamma^2 \sigma_C^2 (1 - \omega^2)} E_t \left[\frac{1 + f^{(i)}(\sigma_{t+1})}{f^{(i+1)}(\sigma_t)} e^{(\sigma_t - \gamma \omega \sigma_C) \varepsilon_{t+1}} \right] \\ + b_0 \rho E_t \left[\hat{v} \left(\frac{1 + f^{(i)}(\sigma_{t+1})}{f^{(i+1)}(\sigma_t)} e^{g_D + \sigma_t \varepsilon_{t+1}} \right) \right], \forall \sigma_t.$$

We continue this process until $f^{(i)} \rightarrow f$.

3.4 Model Results

3.4.1 Calibrating Parameter Values

We calibrate the model at quarterly frequency, such that the model implied moments match those of the observed annual data. In reality, many companies issue their dividend policies and earning reports at quarterly frequency, hence it is reasonable for the investors to re-evaluate their investment performance at a quarterly basis. We also calibrate the model at monthly and annual frequency, in which cases investors re-evaluate their performance more frequently or less frequently. We get qualitatively similar results, so we only report the results based on quarterly decision making throughout our analysis.

Table 3.2 summarizes our choice of parameter values. We choose similar values as BHS for the consumption growth parameters and the preference parameters. For g_C and σ_C , the mean and standard deviation of log consumption growth, we follow Cecchetti, Lam and Mark (1990) and set $g_C = 0.46\%$ and $\sigma_C = 1.90\%$, which corresponds to an annual growth rate of 1.84% with volatility of 3.79%. The curvature γ of utility over consumption and the time discount factor ρ are set as 1.0 and 0.995 respectively, bringing the net annual risk free rate close to 3.86 percent by equation (10) and the values of g_C and σ_C . The loss aversion parameter λ is equal to 2.25, since many independent experimental studies have estimated it as being around this level. Similar to BHS, the parameter b_0 does not have an empirical counterpart, and we present results for a range of values of b_0 .

Using NYSE/AMEX data and Fama risk-free rate data from 1926.Q3 to 2006.Q4 from CRSP, we calibrate the unconditional mean of quarterly dividend growth rate as its empirical mean, $g_D = 0.39\%$. By matching the first moment of Equation (4), $E[\log(|g_{D,t+1} - g_D|)] = \log(\bar{\sigma}) + E[\log(|\varepsilon_{t+1}|)]$, we calibrate $\log(\bar{\sigma})$ as -3.91 . The parameter ϕ , who governs the persistence of dividend volatility, takes the value 0.99, close to the estimated value from an EGARCH model in Section 3.2.

Table 3.2 Calibrated Parameters

This table reports the calibration values for the preference parameters and technology parameters in the theoretical model. γ is the curvature of utility over consumption, ρ is the time discount factor, λ is the loss aversion parameter, and b_0 controls the importance of the loss aversion relative to the consumption in the utility function. g_C and g_D are the means of the consumption and dividend growth rate, respectively. σ_C is the volatility of consumption growth. $\log(\bar{\sigma})$ is the mean of the log of conditional volatility of dividend growth. ϕ measures the persistence of dividend volatility, while σ_u controls the variation in dividend volatility. ω is the correlation between consumption news and dividend news, and θ is the correlation between dividend *level* news and *volatility* news. The calibration for the dividend parameters is based on the dividend sample 1926.Q3-2006.Q3 constructed from the value weighted NYSE/AMEX returns from CRSP.

Parameters	Calibration Values
Preference	
γ	1.0
ρ	0.995
b_0	range
λ	2.25
Technology	
g_C	0.46%
g_D	0.39%
σ_C	1.90%
$\log(\bar{\sigma})$	-3.91
ω	0.15
ϕ	0.99
θ	-0.67
σ_u	0.14

The parameter σ_u is very important since it measures the magnitude of dividend volatility. We calibrate this parameter as 0.14 , such that the model implied annual dividend growth rate has a volatility equal to its empirical counterpart. Compared to BY (2004), the value of σ_u appears large. However, this is an artifact of our specification of the volatility process in equation (5), where the logarithm of dividend volatility rather than its square follows an AR(1) process. Indeed, given $\phi \approx 1$, taking a first order approximation of (5), we have $\sigma_{t+1}^2 \approx \sigma_t^2 + \sigma_w u_{t+1}$, where $\sigma_w = 2E(\sigma_t^2)\sigma_u = 8.2 \times 10^{-4}$, close to the value in BY (2004).

Two more model parameters remain to be calibrated: θ , which captures the interaction between innovations in dividend growth rate and dividend volatility; and ω , the correlation between consumption and dividend. By equations (4) and (5), we calibrate θ at -0.67 . Following Campbell (2000), we set $\omega = 0.15$, which implies an unconditional correlation of 0.1 between consumption and dividend growth processes.

3.4.2 Price-dividend Ratio Function f

Figure 3.4 plots the price-dividend ratio f as a function of $\log(\sigma_t)$ for $b_0 = 0.7$, $b_0 = 2$ and $b_0 = 6$. We also try a variety of other values for b_0 , for example, $b_0 = 0.1$, $b_0 = 20$, $b_0 = 200$, etc. The essential pattern, however, is fully depicted by Figure 3.4.

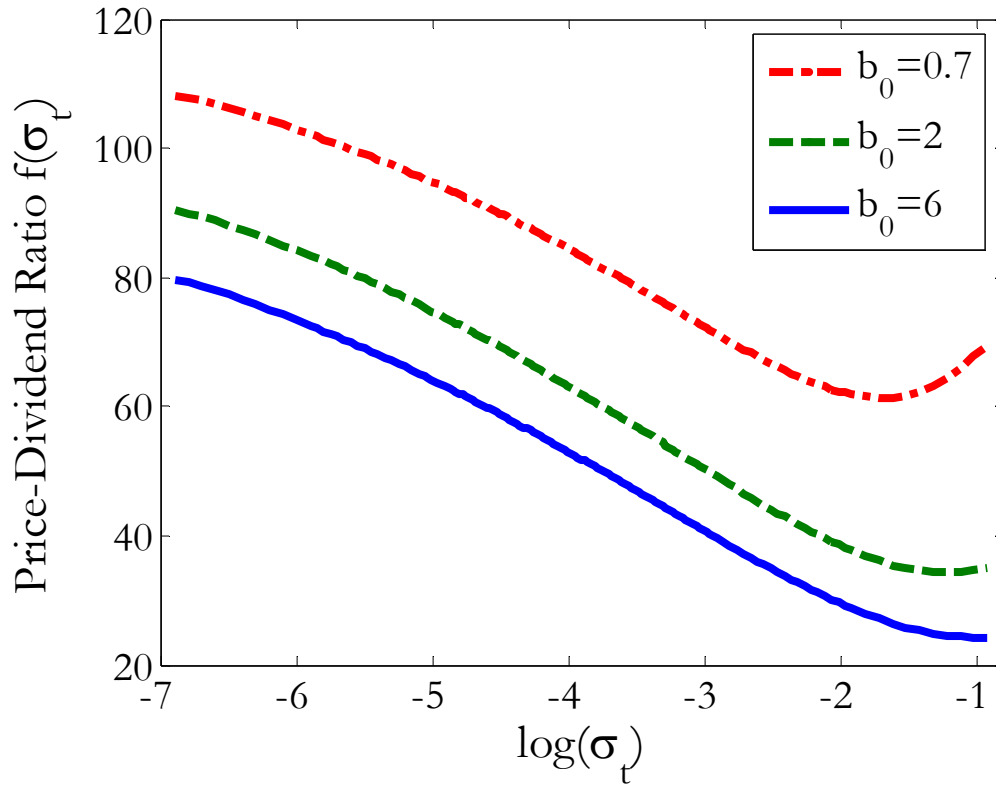


Figure 3.4 Price-Dividend Functions f

Figure 3.4 plots the equilibrium price-dividend ratios against the log of the conditional dividend volatility, $\log(\sigma_t)$, for $b_0 = 0.7, 2$ and 6 .

Investors in our model care not only about consumption, the standard expected log utility term in (2), but also about fluctuations in the value of their investments, the additional prospect utility term in (2). These two forces jointly determine the shape of the function f . Without loss aversion, a higher dividend volatility implies a higher dividend growth rate in the future, and thus higher expected cash flows from holding stocks. Since the stochastic discount factor depends on the consumption process, which is weakly correlated with dividend, it is relatively unchanged. Therefore, stocks are more attractive and their prices are higher. The standard consumption utility contributes to a positive relationship between σ_t and the price-dividend ratio $f(\sigma_t)$.

The presence of loss aversion, in contrast, contributes to a negative relationship between dividend volatility σ_t and the price-dividend ratio $f(\sigma_t)$. For a fixed b_0 , the more volatile the dividend process, the more volatile the returns, therefore, the more likely investors are to suffer financial losses. This causes loss averse investors great pains, and makes stocks less desirable. As a result, they require more compensations when faced with more volatile dividend processes, causing lower stock prices or higher equity premiums.

The negative slope of f function is consistent with BY (2004) and Bansal, Khatchatrian and Yaron (2005), who find that asset prices drop as economic uncertainty rises, although their measure of economic uncertainty is conditional consumption volatility rather than dividend volatility. It is rather difficult to justify this negative relationship within the standard power utility framework, where, as we have discussed before, a higher dividend volatility is associated with higher expected dividend growth, hence price-dividend ratios

always vary positively with dividend volatility. However, it can be easily understood with the introduction of loss aversion preferences.

The overall shape of f can now be summarized as follows. For low values of σ_t , the impact of loss aversion is dominant, hence the function f is downward sloping. As σ_t becomes larger, the impact of log utility catches up, and the function f eventually becomes upward sloping. That is, f is U-shaped, as shown in Figure 3.4. The smaller is b_0 , the earlier f achieves its minimum. Moreover, larger values of b_0 say that investors care more about their wealth fluctuations, in which case the risk premiums for holding stocks are higher. Therefore, as b_0 increases, the function f will move downward.

How does f look like in the data? According to our calibration, $\log(\sigma_t)$ is normally distributed with mean -3.91 and standard deviation 0.99 . Therefore, just reading from Figure 3.4, we will expect to see a negative relationship between price-dividend ratios and dividend volatility for most of the time. To see this more formally, we run an AR(1)-EGARCH(1,1) estimation on the quarterly dividend growth for 1926.Q3-2006.Q4 and plot the price-dividend ratios against the estimated conditional dividend volatilities $\hat{\sigma}_t$ in Figure 3.5. Indeed, more than 80 percent of the observed data display a negative relationship. In addition, we also notice an interesting positive relationship between price-dividend ratios and dividend volatility, which occurs for some extremely high realizations of $\hat{\sigma}_t$. For instance, when the logarithm of dividend volatility is larger than 0.05 , price-dividend ratios actually rise with dividend volatility. Therefore, the data display a similar U-shaped pattern as predicted by our model.

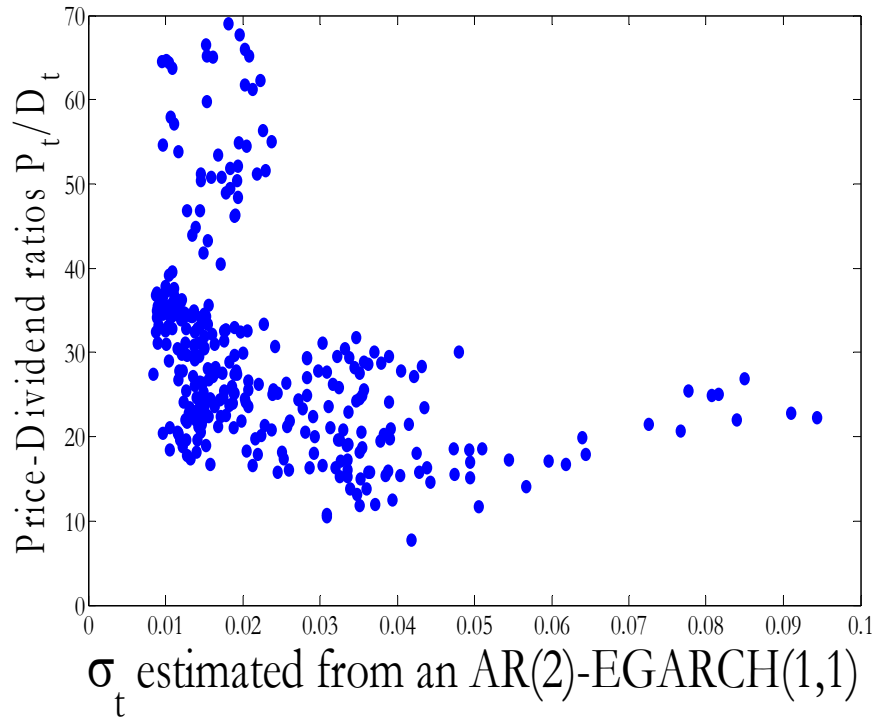


Figure 3.5 Historical Price-Dividend Ratios v.s. Conditional Dividend Volatility

Figure 3.5 plots the historical price-dividend ratios against the conditional dividend volatility estimated from an AR(2)-EGARCH(1,1) regression for period 1926.Q3-2006.Q3.

3.4.3 Simulation Results

In this subsection, we generate artificial data under the parameter configuration in Table 3.2, and show that the model-simulated data replicate the interesting patterns found in actual data.

In order to facilitate a comparison with historical data, we simulate the model at a quarterly frequency and time-aggregate them to get annual data. We do 10,000 simulations each with 320 quarterly observations. We then calculate

the interested statistics and report their sample moments. Given that the simulation number is large enough, the sample moments should serve as good approximations to population moments.

3.4.3.1 Stock Returns and Stock Volatility

Table 3.3 reports a variety of statistics calculated from model simulated data and the corresponding statistics from historical data. It is noteworthy that the model can match the mean and standard deviations of excess stock returns pretty well. When $b_0 = 6$, the model generates a sizable premium of 6.75% per annum, which is slightly higher than the empirical value 5.90%; the model also generates a standard deviation of 19.49%, which is almost equal to the corresponding value of 19.17% in the data.

We notice that as b_0 grows, both the mean and standard deviations from model simulated excess returns increase. This is because when b_0 increases, loss aversion becomes a more important feature of investors' preference, so investors become more and more fearful of risky assets, pushing down stock prices.

We also report the mean and standard deviation of the simulated annual price-dividend ratios, $E(P_t^a / D_t^a)$ and $\sigma(P_t^a / D_t^a)$.⁴⁷ The empirical value $\sigma(P_t^a / D_t^a) = 12.43\%$ is relatively high to those found in other papers (BHS, 2001; BY, 2004; Campbell and Cochrane, 1999). This is due to the relatively high price-dividend ratios from 1996 to 2006, which includes the high-tech bubble

⁴⁷The superscript a indicates *annualized* variables.

period.

Table 3.3 Asset Prices and Annual Returns (1926-2006)

This table provides information regarding stock returns for the simulated data and historical data. The historical data correspond to the period 1926-2006.

The entries for the model are based on 10,000 simulations each with 320 quarterly observations that are time-aggregated to an annual frequency. The parameter configuration in simulation follows that in table 3.2. The expressions $E(r_{t+1}^a - r_{f,t}^a)$ and $\sigma(r_{t+1}^a - r_{f,t}^a)$ are, respectively, the mean and volatility of the annualized continuously compounded returns. $Corr(r_{t+1}^a, g_{C,t+1}^a)$ is the correlation between the annual stock return and annual growth rate. $E(P_t^a / D_t^a)$ and $\sigma(P_t^a / D_t^a)$ are the mean and volatility of the annualized price-dividend ratios.

Variables	Empirical Value (1926-2006)	Model		
		$b_0 = 0.7$	$b_0 = 2$	$b_0 = 6$
Annual Excess Stock Return				
$E\left(r_{t+1}^a - r_{f,t}^a\right)$	5.90	2.68	4.98	6.75
$\sigma\left(r_{t+1}^a - r_{f,t}^a\right)$	19.17	16.18	18.44	19.49
$E\left(r_{t+1}^a - r_{f,t}^a\right) / \sigma\left(r_{t+1}^a - r_{f,t}^a\right)$	0.31	0.16	0.27	0.35
$Corr(r_{t+1}^a, g_{C,t+1}^a)$	0.1	0.1	0.1	0.1
Annual Price-Dividend Ratio				
$E(P_t^a / D_t^a)$	29.08	19.01	13.47	11.06
$\sigma(P_t^a / D_t^a)$	12.43	2.56	2.62	2.47

We are able to match stock returns volatility even though the volatility of price-dividend ratios is lower than their empirical counterparts, a common problem with one factor models. The reason to achieve excess volatility in stock returns is due to the positive relationship between price-dividend ratios and dividend innovations. To see this more clearly, consider the following approximate

relationship (Campbell, Lo and MacKinlay, 1997):

$$r_{t+1} \approx A + \log[f(\sigma_{t+1})/f(\sigma_t)] + \sigma_t \varepsilon_{t+1},$$

where A is a constant. The excess volatility of market return relative to that of the dividend growth (or the fundamental), $Var(r_{t+1}) - Var(\sigma_t \varepsilon_{t+1})$, comes from two sources: the volatility of price-dividend ratios, $Var(\log \frac{f_{t+1}}{f_t})$, and the covariance between the price-dividend ratios and the news to the dividends, $Cov(\log \frac{f_{t+1}}{f_t}, \sigma_t \varepsilon_{t+1})$. In actual data, since $\theta < 0$ in (6), good dividend news (positive ε_{t+1}) tends to be associated with negative dividend volatility shock (negative u_{t+1}), implying that next period price-dividend ratios will increase (see Figure 3.4). Therefore, the covariance term $Cov(\log \frac{f_{t+1}}{f_t}, \sigma_t \varepsilon_{t+1})$ is positive.

The model is also able to generate the low correlation between stock returns and consumption growth, $Corr(r_{t+1}^a, g_{C,t+1}^a) = 0.1$. This happens because the variation in stock returns is completely driven by the innovations in the dividend process, which is only weakly correlated with the consumption process.

3.4.3.2 Autocorrelations of Returns and Price-Dividend Ratios

Table 3.4 presents autocorrelations in returns and price-dividend ratios. Our model predicts negative autocorrelations in stock returns, as documented by Poterba and Summers (1988) and Fama and French (1988a). This negative correlation comes from the fact that returns and price-dividend ratios depend solely on a persistent AR(1) dividend volatility process. Moreover, our model closely matches the highly positively correlated price-dividend ratios in the data.

Table 3.4 Autocorrelations of Returns and Price-Dividend Ratios

This table reports the autocorrelations of annualized stock returns and price-dividend ratios for the simulated data and historical data. The historical data correspond to the period 1926-2006. The entries for the model are based on 10,000 simulations each with 320 quarterly observations that are time-aggregated to an annual frequency. The parameter configuration in simulation follows that in table 3.2. The expressions $Corr(r_t^a, r_{t-j}^a)$ and $Corr(P_t^a/D_t^a, P_{t-j}^a/D_{t-j}^a)$ are, respectively, the autocorrelations of the annualized compound equity returns and P/D ratios.

	Empirical Value	Model		
	(1926-2006)	$b_0 = 0.7$	$b_0 = 2$	$b_0 = 6$
$Corr(r_t^a, r_{t-j}^a)$				
$j = 1$	0.09	-0.01	-0.01	-0.01
$j = 2$	-0.17	-0.02	-0.02	-0.02
$j = 3$	-0.06	-0.02	-0.02	-0.02
$j = 4$	-0.12	-0.02	-0.02	-0.02
$j = 5$	-0.06	-0.02	-0.02	-0.02
$Corr(P_t^a/D_t^a, P_{t-j}^a/D_{t-j}^a)$				
$j = 1$	0.90	0.68	0.79	0.82
$j = 2$	0.81	0.61	0.71	0.74
$j = 3$	0.75	0.56	0.64	0.66
$j = 4$	0.68	0.50	0.58	0.60
$j = 5$	0.60	0.45	0.51	0.53

3.4.3.3 Return Predictability

To analyze the predictability pattern of returns, we run the following regression on both simulated and historical data:

$$r_{t+1}^a + r_{t+2}^a + \dots + r_{t+j}^a = \alpha_j + \beta_j (D_t^a / P_t^a) + \varepsilon_{j,t},$$

where r_{t+j}^a refers to the annual cumulative log returns from year $t + j - 1$ to $t + j$. Table 3.5 presents the regression result for different values of b_0 . This estimation result from model-simulated data resembles the classic pattern documented by Campbell and Shiller (1988) and Fama and French (1988b). The coefficients are significant and negative, indicating that high prices tend to predict low expected returns. Moreover, the forecasting power increases with forecasting horizons, as reflected by the increasing coefficients and R^2 's.

The pattern of return predictability generated by our model can be understood through the volatility test in Cochrane (1992). Starting from the accounting identity $1 = R_{t+1}^{-1} R_{t+1}$ with $R_{t+1} = (P_{t+1} + D_{t+1}) / P_t$, the log-linearization around the average price-dividend ratios, $\overline{P/D}$, implies that, in the absence of rational asset price bubbles,

$$\text{Var}(p_t - d_t) \approx \sum_{j=1}^{\infty} h^j \text{Cov}(p_t - d_t, g_{D,t+j}) - \sum_{j=1}^{\infty} h^j \text{Cov}(p_t - d_t, r_{t+j})$$

where lower case indicates log values and $h = \overline{P/D} / (1 + \overline{P/D})$. This suggests that the variation in the price-dividend ratio will forecast either the change in expected dividend growth rate, or the discount rate, or both.

In our model, even though dividend volatility is time varying, the dividend growth rate *per se* is still a white noise, meaning $\text{Cov}(p_t - d_t, g_{D,t+j}) = 0$. Given that the risk-free rate is maintained as a constant, the only thing remaining for the price-dividend ratio to predict is the excess return. A high price-dividend

ratio is associated with a decline in dividend volatility, so the required expected return will be lower. Therefore, our model implies an extreme version of the volatility test results.

Table 3.5 Return Predictability Regressions (1926-2006)

This table provides evidence of predictability of future excess returns by price-dividend ratios. The entries correspond to regressing $r_{t+1}^a + r_{t+2}^a + \dots + r_{t+j}^a = \alpha_j + \beta_j (D_t^a / P_t^a) + \varepsilon_{j,t}$, where r_{t+j}^a refers to the annual cumulative log returns from year $t+j-1$ to $t+j$. The historical data correspond to the period 1926-2006. The entries for the model are based on 10,000 simulations each with 320 quarterly observations that are time-aggregated to an annual frequency. The parameter configuration in simulation follows that in table 3.2.

	Empirical Value (1926-2006)	Model		
		$b_0 = 0.7$	$b_0 = 2$	$b_0 = 6$
β_1	2.55	2.42	2.08	1.83
β_2	5.99	4.90	4.08	3.58
β_3	8.28	7.23	5.95	5.20
β_4	11.26	9.44	7.71	6.69
$R^2(1)$	0.04	0.03	0.05	0.06
$R^2(2)$	0.09	0.05	0.09	0.11
$R^2(3)$	0.13	0.07	0.13	0.16
$R^2(4)$	0.18	0.09	0.16	0.20

It's worth noting that a central fact driving predictability of returns is the persistence of dividend volatility. As shown in Cochrane (2005), both the estimated coefficients and R^2 's increase with the persistence of the price-dividend ratio, which depends on dividend volatility.

3.4.3.4 Time-varying Sharpe Ratios

Empirical evidence suggests that estimates of both conditional means and conditional standard deviations of returns change through time, but they do not move one for one. Hence Sharpe ratios are time-varying. Figure 3.6 presents the conditional means and conditional standard deviations as functions of the state variable $\log(\sigma_t)$. Overall, as σ_t increases, the dividend growth becomes more volatile; thus both the means and the standard deviations of expected returns increase.

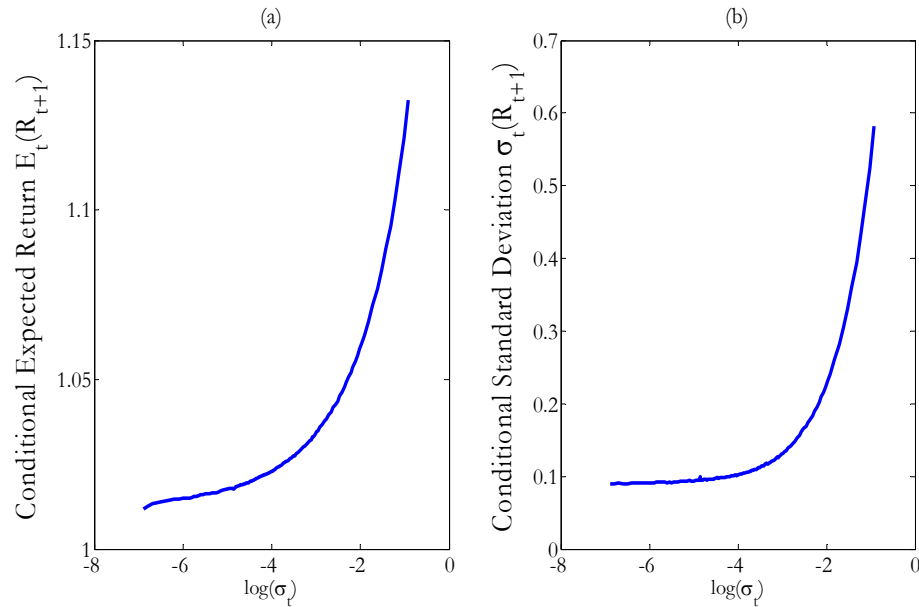


Figure 3.6 Conditional Moments of Stock Returns

Panel (a) and (b) plot the conditional expected stock return $E_t(R_{t+1})$ and conditional volatility of return $\sigma_t(R_{t+1})$ for the case $b_0=6$.

Comparing the conditional means, $E_t(R_{t+1})$, and conditional standard deviations, $\sigma_t(R_{t+1})$, of expected returns, we see that they are different functions of dividend volatility. Most noticeably, for those values of

$\log(\sigma_t)$ smaller than $\log(\bar{\sigma}) = -3.91$, the conditional standard deviation is almost a constant, whereas the conditional mean has more variations and increases with $\log(\sigma_t)$. Therefore, the Sharpe ratio of conditional mean to conditional standard deviation varies over time, with its variation due to the difference between $E_t(R_{t+1})$ and $\sigma_t(R_{t+1})$.

More formally, according to (4)-(7), the conditional mean and conditional variance of R_{t+1} are respectively,

$$E_t(R_{t+1}) = e^{g_D + \frac{1}{2}(1-\theta^2)\sigma_t^2} E_t\left(\frac{1 + f(\sigma_{t+1})}{f(\sigma_t)} e^{\sigma_t \theta u_{t+1}}\right),$$

$$Var_t(R_{t+1}) = e^{2g_D + 2(1-\theta^2)\sigma_t^2} E_t\left[\left(\frac{1 + f(\sigma_{t+1})}{f(\sigma_t)}\right)^2 e^{2\sigma_t \theta u_{t+1}}\right] - e^{2g_D + (1-\theta^2)\sigma_t^2} \left[E_t\left(\frac{1 + f(\sigma_{t+1})}{f(\sigma_t)} e^{\sigma_t \theta u_{t+1}}\right)\right]^2.$$

To get a clearer picture of the distribution of Sharpe ratios, we numerically calculate the conditional Sharpe ratios from the above formula. Specifically, we make 160,000 random draws of ε_{t+1} and u_{t+1} , calculate the conditional mean and conditional standard deviation of expected returns by numerical integration, and then obtain the conditional Sharpe ratios as a function of $\log(\sigma_t)$ when $b_0 = 6$. Figure 3.7 presents the histogram of the simulated conditional Sharpe ratios, showing that the price of risk is changing over time. The unconditional mean and standard deviation of simulated Sharpe ratios are 0.14 and 0.05, matching their empirical values.

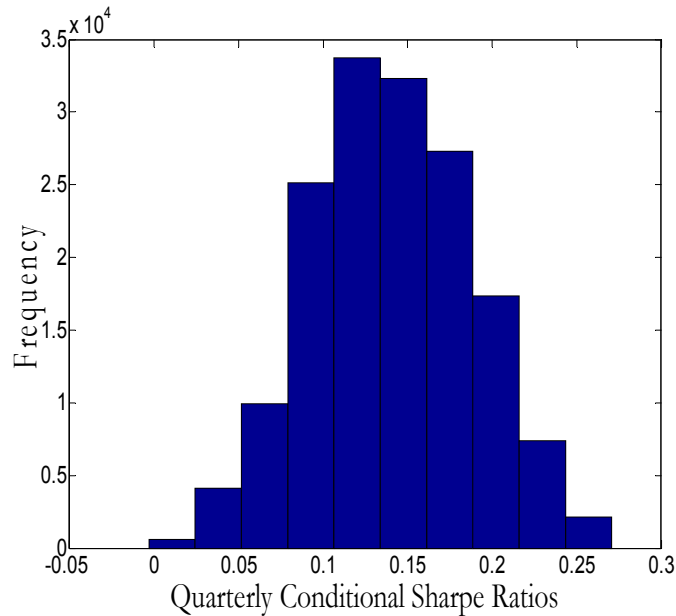


Figure 3.7 Distribution of the Conditional Sharpe Ratios

The distribution is based on a simulation for the case $b_0=6$.

3.4.3.5 Structural Break and Equity Premiums

Recent empirical evidence shows that the macroeconomic risk has declined over the past fifty years. It still remains an open question how this reduced risk affects ex-ante equity premiums, which are identified to have declined since WWII, by Blanchard (1993), Fama and French (2002), Freeman (2004), and Buranavityawut, Freeman and Freeman (2006). We use dividend volatility to stand for risk and study how this risk is priced in financial markets.

The econometric evidence in Section 3.2 suggests that dividend volatility has decreased dramatically since the 1950s. According to our model, lower dividend volatility means that stocks are less likely to perform poorly; thus loss averse investors are less worried about fluctuations in their financial wealth.

As a result, they are more willing to hold risky stocks, pushing up stock prices and driving down expected returns. To test our model performance in the postwar period with declined dividend volatility, we re-calibrate the model according to the data for 1954-2006. The new parameter values are provided in Table 3.6. Comparing the new values with those calibrated from all data, we find that the mean dividend growth rate doesn't change a lot, however, the standard deviation of $\log(\sigma_t)$ decreased from 0.14 to 0.10, a decline of roughly 30%. Consistent with our intuition, the model-simulated data match the actual data very well in excess returns, in the standard deviation of excess returns, as well as in Sharpe ratios.

Our model suggests that the decline in equity premiums is a direct result of declining macroeconomic risk, which is characterized by dividend volatility. The existing literature has focused on other measures of macroeconomic risk. Pastor and Stambaugh (2001) and Kim, Morley and Nelson (2004, 2005) have examined structural changes in market volatility and argue that, if the market price of risk does not vary greatly, then falls in market volatility should be associated with a decline in the required rate of return for equity. Lettau, Ludvigson and Wachter (2008) use consumption volatility to measure economic risk, and argue that this reduced macroeconomic risk contributed to the recent run-up in price-dividend ratios. We prefer dividend volatility to other measures of macroeconomic risk because dividend volatility is an important feature of the underlying endowment process, which determines market volatility in equilibrium. More importantly, as in the data, stock returns are only weakly correlated with consumption, therefore, a model relying on consumption volatility will inevitably generate a high correlation between stock

returns and consumption, contradicting our observation.

Table 3.6 Structural Break and Asset Prices

This table reports the mean and volatility of stock returns for the simulated data for two sets of dividend parameter configurations. Calibration I is based on the dividend sample 1926.Q3-2006.Q3; Calibration II is based on the dividend sample 1954.Q3-2006.Q3. The preference parameters and consumption parameters are the same as table 3.2 for both configurations. The expressions $E(r_{t+1}^a - r_{f,t}^a)$ and $\sigma(r_{t+1}^a - r_{f,t}^a)$ are, respectively, the mean and volatility of the annualized continuously compounded returns.

Dividend Parameter Configuration				
Parameters	σ_u	g_D	$\log(\bar{\sigma})$	θ
Calibration I	0.14	0.39%	-3.91	-0.67
Calibration II	0.10	0.35%	-4.16	-0.34

Annual Excess Returns			
Variables	$E(r_{t+1}^a - r_{f,t}^a)$	$\sigma(r_{t+1}^a - r_{f,t}^a)$	$\frac{E(r_{t+1}^a - r_{f,t}^a)}{\sigma(r_{t+1}^a - r_{f,t}^a)}$
Empirical Value			
1926–2006	5.90	19.17	0.31
1954–2006	4.87	15.35	0.32
Model ($b_0 = 6$)			
Calibration I	6.75	19.49	0.35
Calibration II	4.71	16.54	0.28

3.5 Conclusion

This paper proposes a model in which dividend volatility is used to represent fluctuating economic uncertainty, and investors are loss averse over fluctuations in their financial wealth. Experimental and psychological evidence supports the behavioral assumption of loss aversion. Our empirical analysis of the aggregate dividend (including all distributions) establishes that dividend

volatility is highly persistent and has experienced a remarkable decline in the postwar period.

Our model-simulated data exhibit similar patterns to those observed in actual return data: stock returns have a high mean, high volatility and a low correlation with consumption; they are predicted by price-dividend ratios; the Sharpe ratios are time-varying.

To address the dynamic evolution of equity premiums, we also calibrate the models according to postwar data, in which dividend volatility is shown to be substantially lower than in prewar data. Based on the new calibrated parameter values, the model can generate much lower equity premiums (higher price-dividend ratios) thanks to a more stable economic environment.

In essence, this paper highlights the significant effect of dividend volatility on asset prices when investors are narrow-framing, i.e., they derive direct utility from their financial investments. In the face of an uncertain investment environment captured by dividend volatility, loss averse investors are fearful of holding risky assets; if this uncertainty is persistent, their fears are stronger. This mechanism can generate important asset price behaviors in financial markets.

APPENDIX

We follow Bansal, Khatchatrian and Yaron (2005) to impute the dividend time series from CRSP database. This appendix describes the details. The data covers quarterly sample from 1926.Q3 till 2006.Q3. In order to construct the quarterly dividend variable, the following series are used:

- P_{indx} : Monthly stock price index on NYSE/AMEX. The price index for month j is calculated as $P_{indx,j} = (VWRETX_j + 1) \cdot P_{indx,j-1}$, where $VWRETX$ is the value weighted return on NYSE/AMEX excluding dividends, taken from CRSP.
- D_{indx} : Monthly dividend index on NYSE/AMEX. The dividend for month j is calculated as $D_{indx,j} = \left(\frac{1+VWRETD_j}{1+VWRETX_j} - 1 \right) \cdot P_{indx,j}$, where $VWRETD$ and $VWRETX$ are, correspondingly, the value weighted return on NYSE/AMEX including and excluding dividends, taken from CRSP.
- \bar{D}_{indx} : Quarterly dividend index on NYSE/AMEX. The dividend for a quarter is the sum of the monthly dividend indices for the 3 months comprising the quarter. Then a four period backward moving average is taken to remove seasonality. That is, $\bar{D}_{indx,t} = \frac{1}{4} \sum_{j=0}^3 [D_{indx,3(t-j)-2} + D_{indx,3(t-j)-1} + D_{indx,3(t-j)}]$, where t indexes quarters.
- $Inflation$: Quarterly inflation index. The inflation index for a quarter is the inflation index in the last month of the quarter, taken from CRSP.

The resulting quarterly dividend series and dividend growth rate series are calculated as follows:

$$D_t = \frac{\bar{D}_{indx,t}}{Inflation_t}, g_{D,t} = \log\left(\frac{D_t}{D_{t-1}}\right).$$

REFERENCES

- Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: a potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481-1509.
- Bansal, Ravi, Varoujan Khatchatrian and Amir Yaron, 2005, Interpretable asset markets? *European Economic Review* 49, 531-560.
- Bansal, Ravi, Dana Kiku, and Amir Yaron, 2007, Risks for the long run: estimation and inference, *Working Paper*, Duke University.
- Barberis, Nicholas and Ming Huang, 2001, Mental accounting, loss aversion, and individual stock returns, *Journal of Finance* 56, 1247-1292.
- Barberis, Nicholas and Ming Huang, 2007, Preferences with frames: a new utility specification that allows for the framing of risks, *Working Paper*, Yale University
- Barberis, Nicholas, Ming Huang and Tano Santos, 2001, Prospect theory and asset prices, *Quarterly Journal of Economics* 116, 1-53.
- Benartzi S. and R. Thaler, 1995, Myopic loss aversion and the equity premium puzzle, *Quarterly Journal of Economics* 110, 73-92.
- Blanchard, Olivier J., 1993, Movements in the equity premium, *Brookings Papers on Economic Activity*, 2, 75-118.
- Bollerslev, Timothy, Robert Engle and Jeffrey Wooldridge, 1988, A capital asset pricing model with time-varying covariances, *Journal of Political Economy* 96, 116-131.
- Buranavityawut, Nonthipoth, Mark C. Freeman and Nisih Freeman, 2006, Has the equity premium been low for forty years? *The North American Journal of Economics and Finance*, Volume 17, Issue 2, 191-205.
- Campbell, John Y., Asset Prices, Consumption and the Business Cycle, in

- Handbook of Macroeconomics*, John Taylor and Michael Woodford, eds., Amsterdam: North-Holland, 2000.
- Campbell, John and John Cochrane, 1999, By force of habit: a consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205-251.
- Campbell, John, Andrew Lo and A. Craig MacKinlay, 1997, *The econometrics of financial markets*, Princeton, NJ: Princeton University Press.
- Campbell, John Y., and Robert Shiller, 1988, Stock prices, earnings, and expected dividends, *Journal of Finance*, XLIII, 661--676.
- Cecchetti, Stephen, Pok-sang Lam, and Nelson Mark, 1990, Mean reversion in equilibrium asset prices, *American Economic Review*, LXXX, 398--418.
- Cochrane, John, 1992, Explaining the variance of price-dividend ratios, *Review of Financial Studies*, 5, 243-280.
- Cochrane, John, 2005, *Asset Pricing*. Princeton University Press, Princeton, NJ.
- Constantinides, George, and Darrell Duffie, 1996, Asset pricing with heterogeneous consumers, *Journal of Political Economy*, 104, 219-240.
- Espstein, Lawrence and Stanley Zin, 1989, Substitution, risk aversion, and the temporal behavior of asset returns: a theoretical framework, *Econometrica* 57, 937-969.
- Fama, Eugene, and Kenneth French, 1988a, Permanent and temporary components of stock prices, *Journal of Political Economy*, XCVI, 246--273.
- Fama, Eugene, and Kenneth French, 1988b, Dividend yields and expected stock returns, *Journal of Financial Economics*, XXII, 3--25.
- Fama, Eugene and Kenneth French, 2002, The equity premium, *Journal of Finance*, 57, 637-659.

- Freeman M. C., 2004, An explanation of the declining ex-ante equity premium, *Working Paper*, University of Exeter.
- Hamilton, J. D., 1989, A new approach to the analysis of nonstationary returns and the business cycle, *Econometrica*, 57, 357-384.
- Hansen, Lars P., and Kenneth Singleton, 1983, Stochastic consumption, risk aversion and the temporal behavior of asset returns, *Journal of Political Economy*, XCI, 249--265.
- Heaton, John and Deborah Lucas, 1996, Evaluating the effects of incomplete markets on risk sharing and asset pricing, *Journal of Political Economy*, 104, 668-712.
- Kahneman, Daniel, 2003, Maps of bounded rationality: psychology for behavioral economics, *American Economic Review* 93, 1449-75.
- Kahneman, Daniel, and Dan Lovallo, 1993, Timid choices and bold forecasts: a cognitive perspective on risk taking, *Management Science* 39, 17-31.
- Kahneman, Daniel, and Amos Tversky, 1979, Prospect theory: an analysis of decision under risk, *Econometrica* 47, 263-291.
- Kim, Chang-Jin, James Morley and Charles Nelson, 2004, Is there a positive relationship between stock market volatility and the equity premium? *Journal of Money, Credit and Banking*, 36, 339-360.
- Kim, Chang-Jin, James Morley and Charles Nelson, 2005, The structural break in the equity premium, *Journal of Business and Economic Statistics*, 23, 181-191.
- Lettau, Martin, Sydney Ludvigson and Jessica Wachter, 2008, The declining risk premium: what role does macroeconomic risk play? *The Review of Financial Studies*.
- Longstaff, Francis and Monika Piazzesi, 2004, Corporate earnings and the

equity premium, *Journal of Financial Economics*, 74, 401-421.

Pastor, Lubos, and Robert Stambaugh, 2001, The Equity Premium and Structural Breaks, *Journal of Finance* 56, 1207--1239.

Poterba, James, and Lawrence Summers, 1988, Mean-reversion in stock returns: evidence and implications, *Journal of Financial Economics*, XXII, 27--60.